

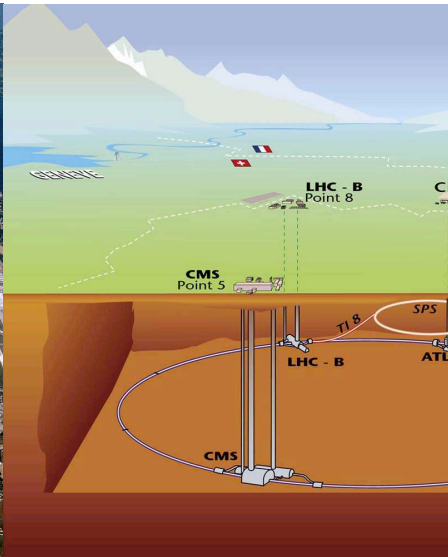
Potential Discoveries at the Large Hadron Collider

Chris Quigg

Fermilab

quigg@fnal.gov

LHC is operating, breaking new ground in E & \mathcal{L}



Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

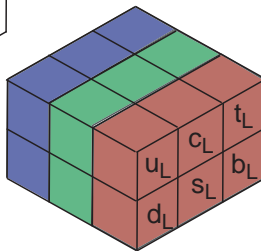
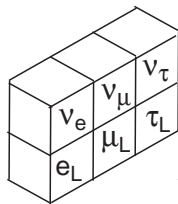
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

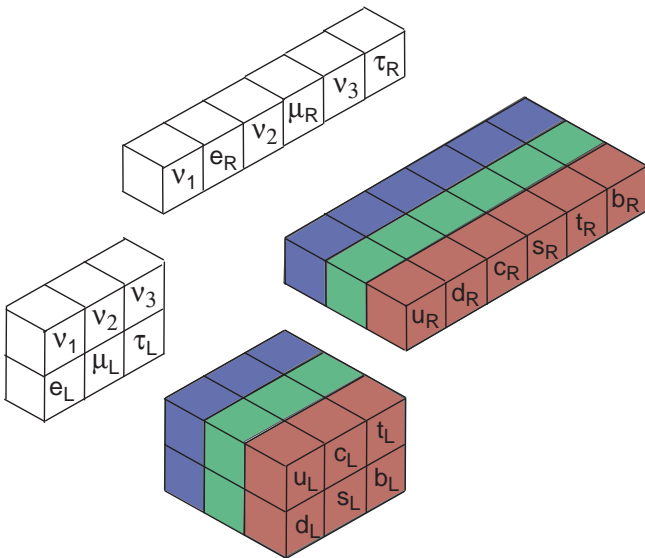
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Few fundamental forces, derived from gauge symmetries

$$\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$$

Electroweak symmetry breaking: Higgs mechanism?





Symmetries \implies interactions: Phase Invariance in QM

QM state: complex Schrödinger wave function $\psi(x)$

Observables $\langle O \rangle = \int d^n x \psi^* O \psi$ are *unchanged*

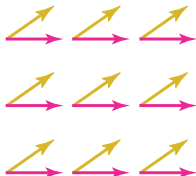
under a global phase rotation

$$\begin{aligned}\psi(x) &\rightarrow e^{i\theta} \psi(x) \\ \psi^*(x) &\rightarrow e^{-i\theta} \psi^*(x)\end{aligned}$$

- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

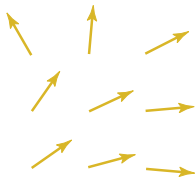


Global rotation — same everywhere



Might we choose one phase convention in Tainan and another in Batavia?

A different convention at each point?



$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

There is a price ...

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation $\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$, the gradient of the wave function transforms as

$$\nabla\psi(x) \rightarrow e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)].$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

To restore local phase invariance, modify eqns. of motion, observables.

Replace ∇ by $\nabla + iq\vec{A}$	“Gauge-covariant derivative”
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If the vector potential \vec{A} transforms under local phase rotations as

$$\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \rightarrow e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

Note ...

- $\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- Replacement $\nabla \rightarrow (\nabla + iq\vec{A})$ corresponds to $\vec{p} \rightarrow \vec{p} - q\vec{A}$

Form of interaction deduced from local phase invariance

Maxwell's equations derived from a symmetry principle

QED is the gauge theory based on $U(1)$ phase symmetry

General procedure ... also in field theory

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
(Connection with conservation laws)
- Symmetry in stricter (local) form \leadsto interactions.

Results in ...

- Massless vector fields (gauge fields).
- Minimal coupling to the conserved current.
- Interactions among gauge fields, if non-Abelian.

Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical \mathcal{L} ; consistent quantum theory may require additional vigilance). Formalism is no guarantee that the gauge symmetry was chosen wisely.

Phase invariance in field theory

Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

for a free fermion follows from the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x),$$

where $\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$, on applying Euler–Lagrange equations,

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi(x))}.$$

Impose local phase invariance:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - qA_\mu \bar{\psi}\gamma^\mu \psi \\ &= \mathcal{L}_{\text{free}} - J^\mu A_\mu,\end{aligned}$$

where $J^\mu = q\bar{\psi}\gamma^\mu\psi$ (follows from global phase invariance)

Problem 1

Verify that the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi$$

is invariant under the combined transformations

$$\begin{aligned}\psi(x) &\rightarrow e^{iq\alpha(x)}\psi(x) \\ A_\mu(x) &\rightarrow A_\mu(x) - \partial_\mu\alpha(x).\end{aligned}$$

Toward QED

Add kinetic energy term for the vector field, to describe the propagation of free photons.

$$\mathcal{L}_\gamma = -\frac{1}{4}(\partial_\nu A_\mu - \partial_\mu A_\nu)(\partial^\nu A^\mu - \partial^\mu A^\nu).$$

Assembling the pieces: $\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{free}} - J^\mu A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

A photon mass term would have the form

$$\mathcal{L}_\gamma = \frac{1}{2}M_\gamma^2 A^\mu A_\mu,$$

which obviously violates local gauge invariance because

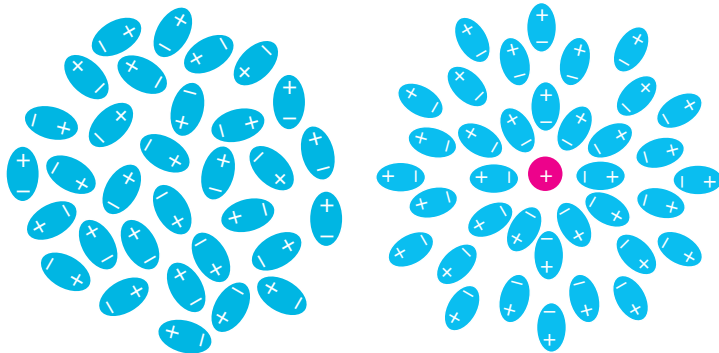
$$A^\mu A_\mu \rightarrow (A^\mu - \partial^\mu \alpha)(A_\mu - \partial_\mu \alpha) \neq A^\mu A_\mu.$$

Local gauge invariance \leadsto massless photon: observe $M_\gamma < 10^{-18} \text{ eV}/c^2$

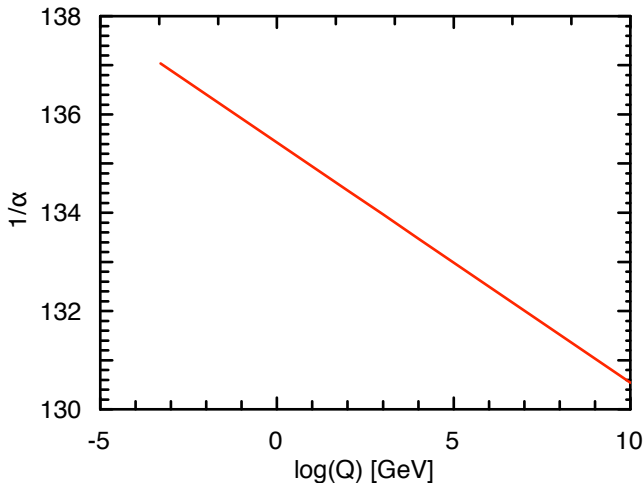
arXiv:0809.1003

Charge screening in electrodynamics

Dielectric (polarizable) medium ...

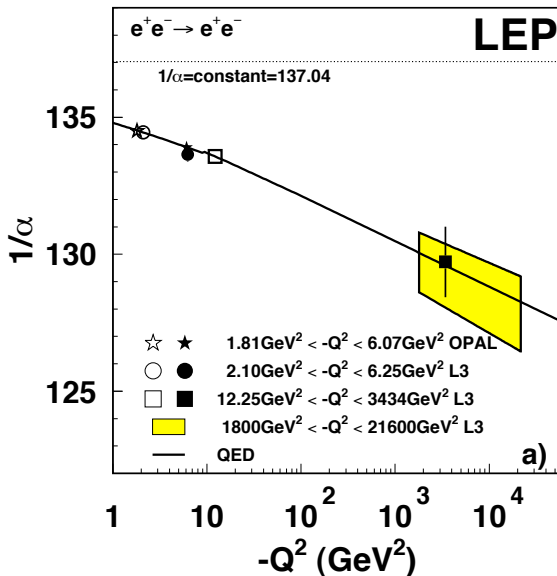


Charge screening in QED (electrons + photons)



$$1/\alpha(Q) = 1/\alpha_0 - \frac{2}{3\pi} \ln \left(\frac{Q}{m} \right)$$

Charge screening in QED (real world)



Non-Abelian Gauge Theories

Free-nucleon Lagrangian (for composite fermion fields)

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

$$\psi \equiv \begin{pmatrix} p \\ n \end{pmatrix}$$

Invariant under global isospin $\psi \rightarrow \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\alpha}/2)\psi$
conserved isospin current $\mathbf{J}^\mu = \bar{\psi}\gamma^\mu\frac{\boldsymbol{\tau}}{2}\psi$.

Local isospin invariance?

Non-Abelian Gauge Theories ...

Under a local gauge transformation,

$$\psi(x) \rightarrow \psi'(x) = \mathcal{G}(x)\psi(x),$$

$$\text{with } \mathcal{G}(x) \equiv \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\alpha}(x)/2),$$

gradient transforms as

$$\partial_\mu \psi \rightarrow \mathcal{G}(\partial_\mu \psi) + (\partial_\mu \mathcal{G})\psi.$$

Introduce a gauge-covariant derivative

$$\mathcal{D}_\mu \equiv I\partial_\mu + igB_\mu \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Non-Abelian Gauge Theories ...

2×2 matrix defined by

$$B_\mu = \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{b}_\mu = \frac{1}{2} \tau^a b_\mu^a = \frac{1}{2} \begin{pmatrix} b_\mu^3 & b_\mu^1 - ib_\mu^2 \\ b_\mu^1 + ib_\mu^2 & -b_\mu^3 \end{pmatrix}$$

gauge fields $\mathbf{b}_\mu = (b_\mu^1, b_\mu^2, b_\mu^3)$, isospin index $a = 1 \dots 3$.

Require $\mathcal{D}_\mu \psi \rightarrow \mathcal{D}'_\mu \psi' = G(\mathcal{D}_\mu \psi)$ to learn how B_μ must behave under gauge transformations.

$$b'^\ell_\mu = b^\ell_\mu - \varepsilon_{jkl} \alpha^j b^k - \frac{1}{g} \partial_\mu \alpha^\ell$$

Transformation rule depends on ε_{jkl} not on representation

Adding a kinetic term for gauge bosons

So far, free Dirac Lagrangian plus interaction coupling isovector gauge fields to conserved isospin current.

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi \\ &= \mathcal{L}_0 - g\bar{\psi}\gamma^\mu B_\mu\psi \\ &= \mathcal{L}_0 - \frac{g}{2}\mathbf{b}_\mu \cdot \bar{\psi}\gamma^\mu \boldsymbol{\tau}\psi,\end{aligned}$$

Copying QED for field-strength tensor doesn't work

$$\partial_\nu B'_\mu - \partial_\mu B'_\nu \neq G(\partial_\nu B_\mu - \partial_\mu B_\nu)G^{-1}.$$

Could write QED case as

$$F_{\mu\nu} = \frac{1}{iq} [\mathcal{D}_\nu, \mathcal{D}_\mu]$$

Adding a kinetic term for gauge bosons

Candidate for SU(2):

$$\mathcal{F}_{\mu\nu} = \frac{1}{ig} [\mathcal{D}_\nu, \mathcal{D}_\mu] = \partial_\nu B_\mu - \partial_\mu B_\nu + iq [B_\nu, B_\mu]$$

transforms as required!

$$\mathcal{L}_{\text{YM}} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \frac{1}{2}\text{tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

invariant under local gauge transformations

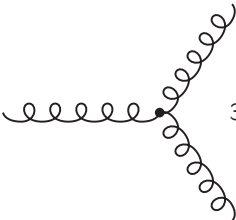
Gauge-boson mass $M^2 B_\mu B^\mu$ not gauge invariant;
common nucleon mass $m\bar{\psi}\psi$ allowed.

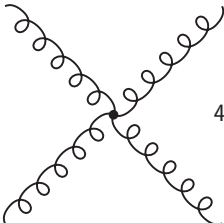
In component form, $\mathcal{F}_{\mu\nu}^I = \partial_\nu b_\mu^I - \partial_\mu b_\nu^I + g\varepsilon_{jkl}b_\mu^j b_\nu^k$
general gauge group, $\varepsilon_{jkl} \rightsquigarrow f_{jkl}$

Gauge-boson self-interactions from $\frac{1}{2}\text{tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$

SU(2):

 gauge-boson propagator

 3-gauge-boson vertex

 4-gauge-boson vertex

Quantum Chromodynamics: Yang-Mills theory for $SU(3)_c$

Single quark flavor:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \frac{1}{2}\text{tr}(G_{\mu\nu} G^{\mu\nu})$$

composite spinor for color-**3** quarks of mass m

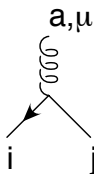
$$\psi = \begin{pmatrix} q_{\text{red}} \\ q_{\text{green}} \\ q_{\text{blue}} \end{pmatrix}$$

Gauge-covariant derivative:

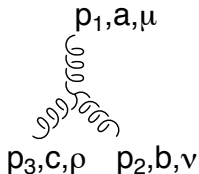
$$\mathcal{D}_\mu = \partial_\mu + igB_\mu$$

g : strong coupling; B_μ : 3×3 matrix in color space formed from 8 gluon fields B_μ^ℓ and $SU(3)_c$ generators $\frac{1}{2}\lambda^\ell \dots$

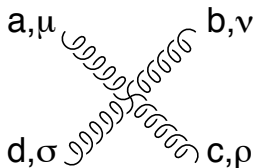
QCD ...



$$-ig \gamma^\mu T_{ij}^a$$



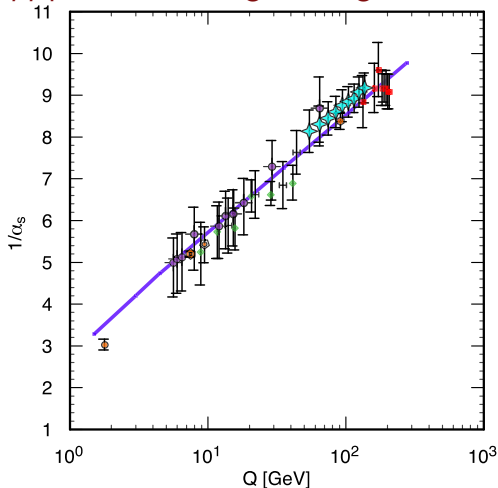
$$-gf^{abc} \left((p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\mu\rho} \right)$$



$$\begin{aligned} & -ig^2 f^{abe} f^{cde} (g^{\nu\sigma} g^{\mu\rho} - g^{\mu\sigma} g^{\nu\rho}) \\ & -ig^2 f^{ace} f^{bde} (g^{\rho\sigma} g^{\mu\nu} - g^{\mu\sigma} g^{\nu\rho}) \\ & -ig^2 f^{ade} f^{cbe} (g^{\nu\sigma} g^{\mu\rho} - g^{\rho\sigma} g^{\mu\nu}) \end{aligned}$$

Color *antiscreening* in QCD

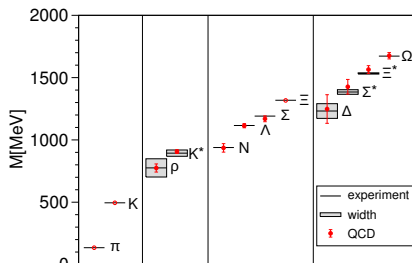
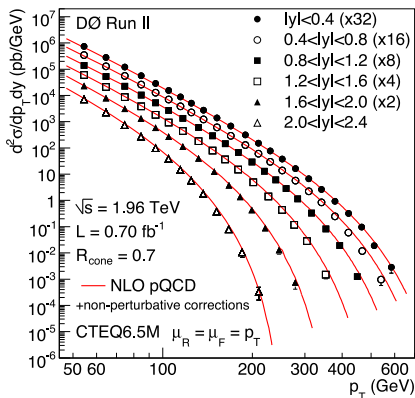
Screening from $q\bar{q}$ pairs, camouflage from gluon cloud



$$\frac{1}{\alpha_s(Q)} = \frac{1}{\alpha_s(\mu)} + \frac{(33 - 2n_f)}{6\pi} \ln \left(\frac{Q}{\mu} \right)$$

Asymptotic Freedom: α_s decreases at large Q

- \rightsquigarrow domain in which strong-interaction processes may be treated perturbatively
- *Infrared slavery* at long distances \rightsquigarrow confinement of quarks into color-singlet hadrons



Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry

A theory of leptons

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad R \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$ gauge group \Rightarrow gauge fields:

- weak isovector \vec{b}_μ , coupling g

$$b_\mu^\ell = b_\mu^\ell - \varepsilon_{jkl} \alpha^j b_\mu^k - (1/g) \partial_\mu \alpha^\ell$$

- weak isoscalar \mathcal{A}_μ , coupling $g'/2$

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \partial_\mu \alpha$$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{jkl} b_\mu^j b_\nu^k, \text{ } SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu, \text{ } U(1)_Y$$

Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}$$

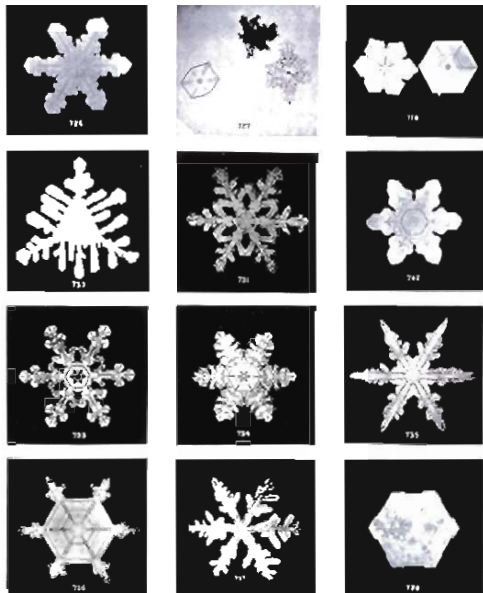
$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^{\ell}F^{\ell\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

$$\begin{aligned}\mathcal{L}_{\text{leptons}} = & \bar{R} i\gamma^{\mu} \left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y \right) R \\ & + \bar{L} i\gamma^{\mu} \left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_{\mu} \right) L.\end{aligned}$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e}e$ violates local gauge inv.

Theory: 4 massless gauge bosons (\mathcal{A}_{μ} b_{μ}^1 b_{μ}^2 b_{μ}^3); Nature: 1 (γ)

Symmetry of laws \nRightarrow symmetry of outcomes



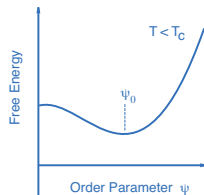
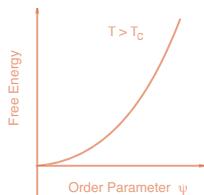
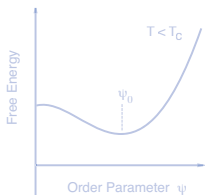
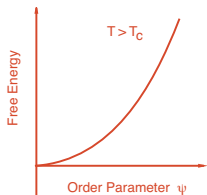
Massive Photon? *Hiding Symmetry*

Recall **2** miracles of superconductivity:

- No resistance Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, **resistive** charge carriers + superconducting charge carriers



$$\mathbf{B} = 0: \quad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0$$

In a nonzero magnetic field ...

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left. \begin{matrix} e^* = -2 \\ m^* \end{matrix} \right\} \text{ of superconducting carriers}$$

Weak, slowly varying field: $\psi \approx \psi_0 \neq 0$, $\nabla\psi \approx 0$

Variational analysis \leadsto wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass

$$\lambda^{-1} = e^* |\langle\psi\rangle_0| / \sqrt{m^* c^2}$$

within superconductor

origin of Meissner effect

Magnet floats (on field lines) above superconductor



Meissner effect levitates Leon Lederman (Snowmass 2001)



Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to \mathcal{L} (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2$$

- Add a Yukawa interaction $\mathcal{L}_{\text{Yukawa}} = -\zeta_e [\bar{R}(\phi^\dagger L) + (\bar{L}\phi)R]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{em}$ invariance

Invariance under \mathcal{G} means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

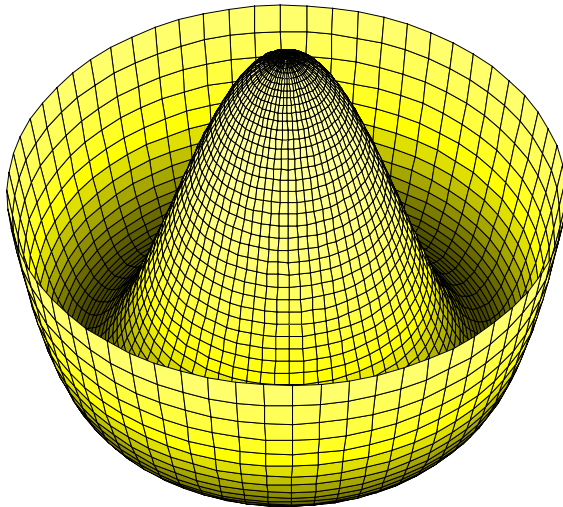
$$\tau_1 \langle \phi \rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2 \langle \phi \rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3 \langle \phi \rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y \langle \phi \rangle_0 = Y_\phi \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws \nRightarrow symmetry of outcomes



Examine electric charge operator Q on the (neutral) vacuum

$$\begin{aligned} Q\langle\phi\rangle_0 &= \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\ &= \frac{1}{2} \begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix} \langle\phi\rangle_0 \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{unbroken!} \end{aligned}$$

Four original generators are broken, *electric charge is not*

- $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$\tau_1, \tau_2, \frac{1}{2}(\tau_3 - Y) \equiv K \quad \text{to acquire masses}$$

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &\quad + \frac{v^2}{8}[g^2 |b_\mu^1 - ib_\mu^2|^2 + (g' \mathcal{A}_\mu - gb_\mu^3)^2] \\ &\quad + \text{interaction terms}\end{aligned}$$

“Higgs boson” η has acquired (mass)² $M_H^2 = -2\mu^2 > 0$

$$\text{Define } W_\mu^\pm = \frac{b_\mu^1 \mp ib_\mu^2}{\sqrt{2}}$$

$$\frac{g^2 v^2}{8}(|W_\mu^+|^2 + |W_\mu^-|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

$$(v^2/8)(g'\mathcal{A}_\mu - gb_\mu^3)^2 \dots$$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + gb_\mu^3}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g\mathcal{A}_\mu + g'b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2} v/2 = M_W \sqrt{1 + g'^2/g^2}$$

A_μ remains massless

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v + \eta)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R) \\
&= -\frac{\zeta_e v}{\sqrt{2}} \bar{e} e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e} e
\end{aligned}$$

electron acquires $m_e = \zeta_e v / \sqrt{2}$

Higgs-boson coupling to electrons: m_e/v (\propto mass)

Desired particle content ... plus a Higgs scalar

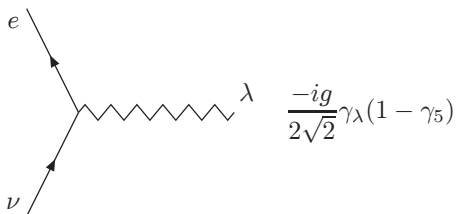
Values of couplings, electroweak scale v ?

What about interactions?

Interactions ...

$$\mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

+ similar terms for μ and τ



W

$$\text{zigzag line} = \frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2}.$$

Compute $\nu_\mu e \rightarrow \mu \nu_e$

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

$$\Rightarrow \langle \phi^0 \rangle_0 = (G_F \sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W -propagator modifies HE behavior

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

Partial-wave unitarity respected for

$$s < M_W^2 [\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

Interactions ...

$$\mathcal{L}_{A-\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu \quad \text{vector interaction}$$

$$\leadsto A_\mu \text{ as } \gamma, \text{ provided } \boxed{gg' / \sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan \theta_W$ θ_W : weak mixing angle

$$g = e / \sin \theta_W \geq e$$

$$g' = e / \cos \theta_W \geq e$$

$$Z_\mu = b_\mu^3 \cos \theta_W - \mathcal{A}_\mu \sin \theta_W \quad A_\mu = \mathcal{A}_\mu \cos \theta_W + b_\mu^3 \sin \theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu \quad \text{LH}$$

Interactions ...

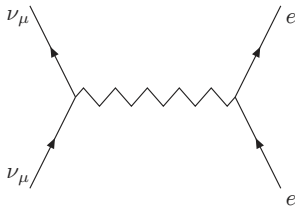
$$\mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] e Z_\mu$$

$$L_e = 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3$$

$$R_e = 2 \sin^2 \theta_W = 2x_W$$

Neutral-current interactions

New $\nu_\mu e$ reaction:



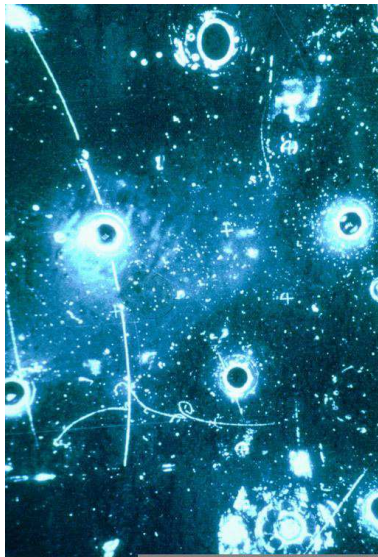
$$\sigma(\nu_\mu e \rightarrow \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2 + R_e^2/3]$$

$$\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2/3 + R_e^2]$$

$$\sigma(\nu_e e \rightarrow \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2 + R_e^2/3]$$

$$\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2/3 + R_e^2]$$

Gargamelle $\nu_\mu e$ event (1973)



Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} \left[\bar{u} \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W-\ell}$: universality \Leftrightarrow weak isospin

- NC interaction

$$\mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu [L_i(1 - \gamma_5) + R_i(1 + \gamma_5)] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z-\ell}$

Electroweak Theory: First Assessment

- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha / G_F \sqrt{2} \sin^2 \theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2 / \cos^2 \theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2 \theta_W$ to predict M_W, M_Z

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass,
but partial-wave unitarity defines tipping point

Gedanken experiment: high-energy scattering of

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad HZ_L^0$$

L : longitudinal, $1/\sqrt{2}$ for identical particles

The importance of the 1-TeV scale . .

In HE limit, s -wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect partial-wave unitarity condition $|a_0| \leq 1$

$$\Rightarrow M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}$$

condition for perturbative unitarity

The importance of the 1-TeV scale . . .

If the bound is respected

- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among W^\pm , Z , H become strong on 1-TeV scale

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Tevatron: $\bar{p}p$ at $\sqrt{s} = 1.96$ TeV



Unanswered Questions in the Electroweak Theory

Chris Quigg

Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, Illinois 60510
Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
Theory Group, Physics Department, CERN, CH-1211 Geneva 23, Switzerland

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Key Words

electroweak symmetry breaking, Higgs boson, 1-TeV scale, Large Hadron Collider (LHC), hierarchy problem, extensions to the Standard Model

Abstract

This article is devoted to the status of the electroweak theory on the eve of experimentation at CERN's Large Hadron Collider (LHC). A compact summary of the logic and structure of the electroweak theory precedes an examination of what experimental tests have established so far. The outstanding unconfirmed prediction is the existence of the Higgs boson, a weakly interacting spin-zero agent of electroweak symmetry breaking and the giver of mass to the weak gauge bosons, the quarks, and the leptons. General arguments imply that the Higgs boson or other new physics is required on the 1-TeV energy scale.

Even if a "standard" Higgs boson is found, new physics will be implicated by many questions about the physical world that the Standard Model cannot answer. Some puzzles and possible resolutions are recalled. The LHC moves experiments squarely into the 1-TeV scale, where answers to important outstanding questions will be found.

Electroweak theory antecedents

Lessons from experiment and theory

- Parity-violating $V - A$ structure of charged current
- Cabibbo universality of leptonic and semileptonic processes
- Absence of strangeness-changing neutral currents
- Negligible neutrino masses; left-handed neutrinos
- Unitarity: four-fermion description breaks down at $\sqrt{s} \approx 620 \text{ GeV}$ $\nu_\mu e \rightarrow \mu \nu_e$
- $\nu\bar{\nu} \rightarrow W^+W^-$: divergence problems of *ad hoc* intermediate vector boson theory

Electroweak theory consequences

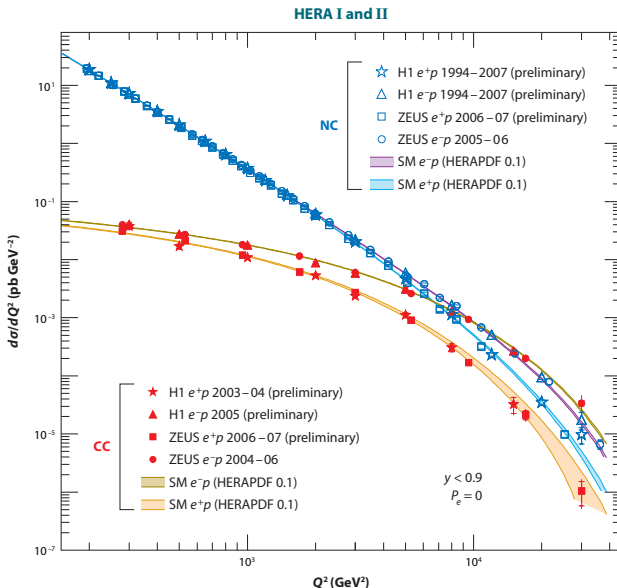
- Weak neutral currents
- Need for charmed quark
- Existence and properties of W^{\pm} , Z^0
- No flavor-changing neutral currents at tree level
- No right-handed charged currents
- CKM Universality
- KM phase dominant source of CP violation
- Existence and properties of Higgs boson
- Higgs interactions determine fermion masses, *but ...*
- (Massless neutrinos: no neutrino mixing)

Electroweak theory tests: tree level

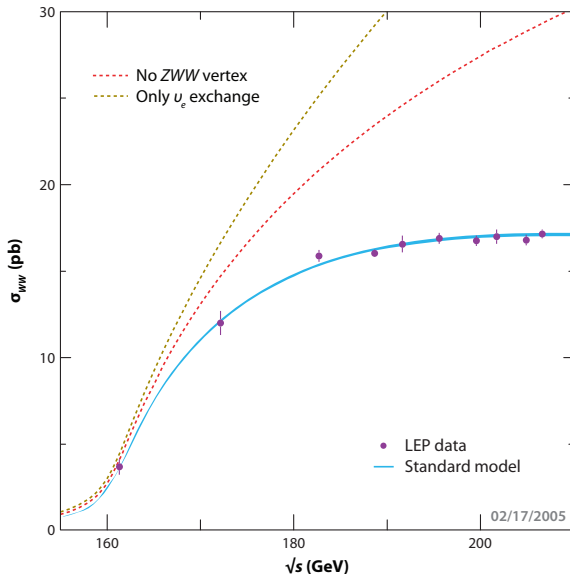
- W^\pm, Z^0 existence and properties verified
- Z -boson chiral couplings to quarks and leptons agree with $SU(2)_L \otimes U(1)_Y$ theory
- Third generation of quarks and leptons discovered
- Constraints on a fourth generation
- $M_{Z'} \gtrsim 789$ GeV (representative cases)
- $M_{W'} \gtrsim 1000$ GeV
- $M_{W_R} \gtrsim 715$ GeV, $g_L = g_R$
- Strong suppression of FCNC:

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.73_{-1.05}^{+1.15} \times 10^{-10};$$
$$\text{SM expectation} = (0.85 \pm 0.07) \times 10^{-10}$$

Electroweak theory tests: tree level

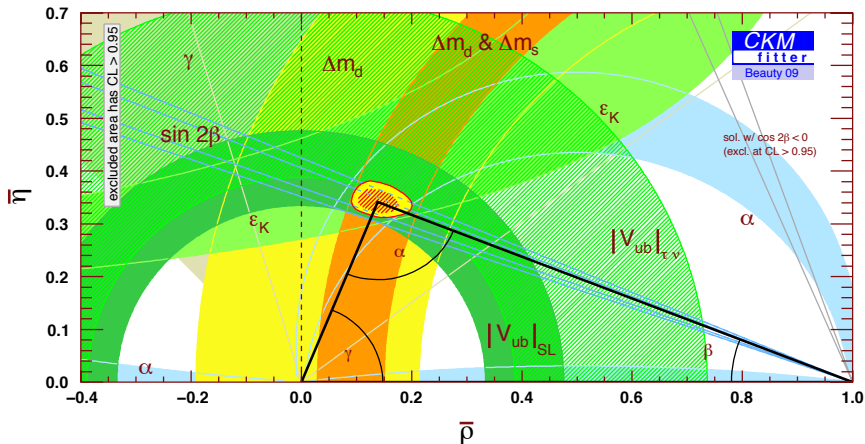


Electroweak theory tests: tree level

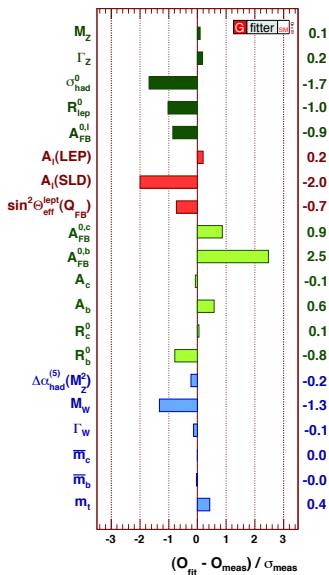


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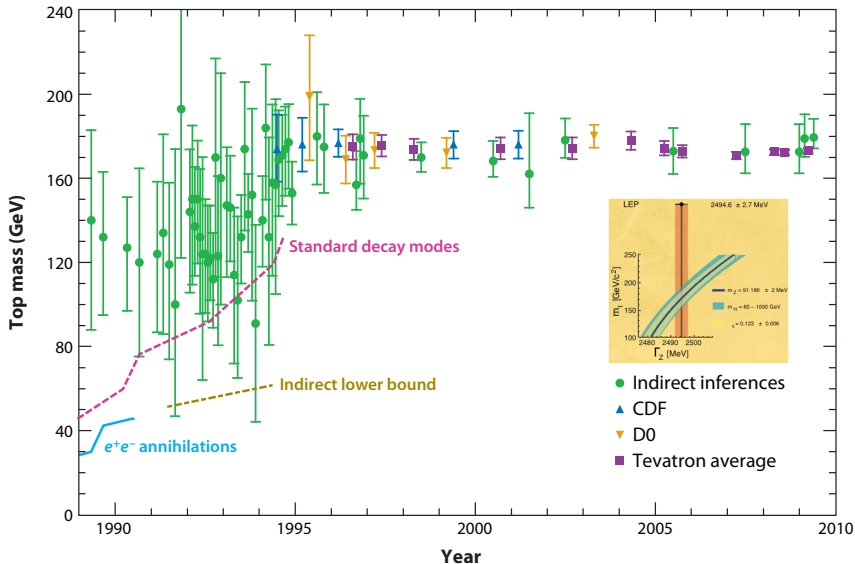
Electroweak theory tests: CKM paradigm



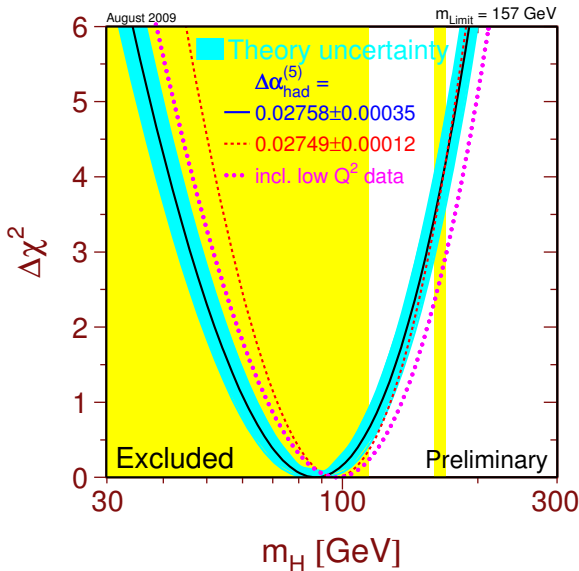
Electroweak theory tests: loop level



Electroweak theory tests: loop level



Electroweak theory tests: Higgs influence



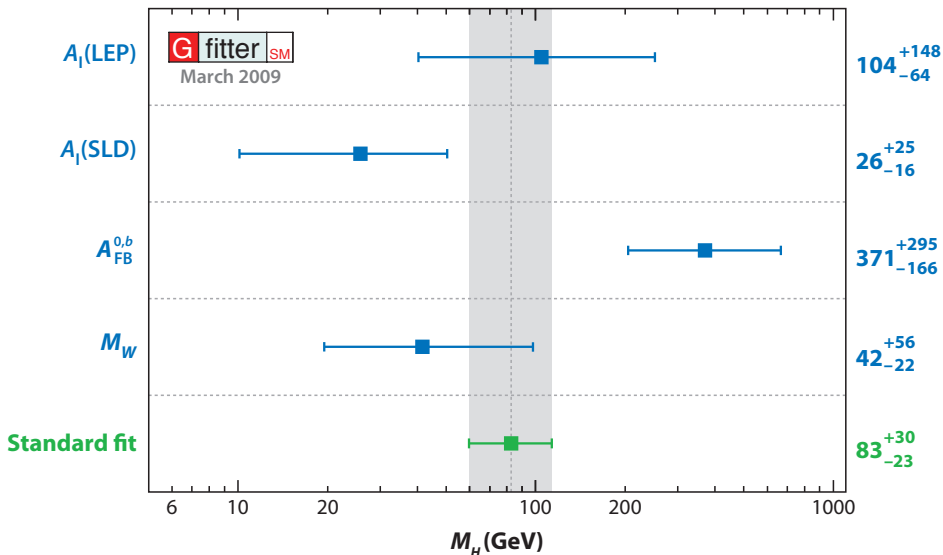
A Cautionary Note

- A_{FB}^b , which exerts the greatest “pull” on the global fit, is most responsible for raising M_H above the range excluded by direct searches.
- Leptonic and hadronic observables point to different best-fit values of M_H
- Many subtleties in experimental and theoretical analyses

M. Chanowitz, [arXiv:0806.0890](https://arxiv.org/abs/0806.0890)

Introduction to global analyses: J. L. Rosner, [hep-ph/0108195](https://arxiv.org/abs/hep-ph/0108195);
[hep-ph/0206176](https://arxiv.org/abs/hep-ph/0206176)

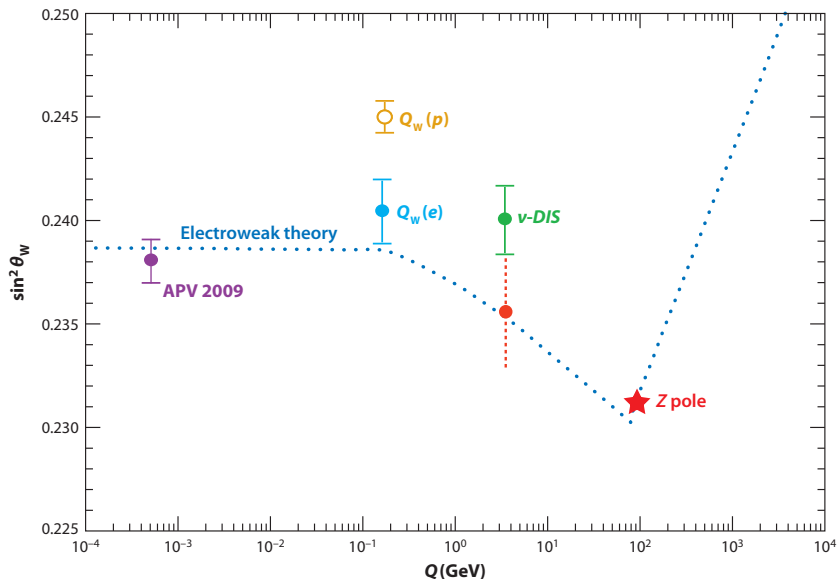
Electroweak theory tests: Higgs consistency?



M_H for individual sensitive observables

Electroweak theory tests: low scales

[Z']



Electroweak theory successes

↪ search for agent of EWSB

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Rep. Prog. Phys. **70** (2007) 1019–1053

[doi:10.1088/0034-4885/70/7/R01](https://doi.org/10.1088/0034-4885/70/7/R01)

Spontaneous symmetry breaking as a basis of particle mass

Chris Quigg

Higgs (then)



Kibble, Guralnik, Hagen, Englert, Brout (now)



What the LHC *is not* really for ...

- Find the Higgs boson,
the Holy Grail of particle physics,
the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

We are not ticking off items on a shopping list ...

We are exploring a vast new terrain
...and reaching the Fermi scale



Electroweak Questions for the LHC

- What hides electroweak symmetry: a Higgs boson, or new strong dynamics?
- If a Higgs boson: one or several?
- Elementary or composite?
- Is the Higgs boson indeed light, as anticipated by the global fits to EW precision measurements?
- Does H only give masses to W^\pm and Z^0 , or also to fermions? (Infer $t\bar{t}H$ from production)
- Are the branching fractions for $f\bar{f}$ decays in accord with the standard model?

If all this: what sets the fermion masses and mixings?

Search for the Standard-Model Higgs Boson

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass; $\propto \beta^3$ for scalar

$$\Gamma(H \rightarrow W^+ W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2) \quad x \equiv 4M_W^2/M_H^2$$

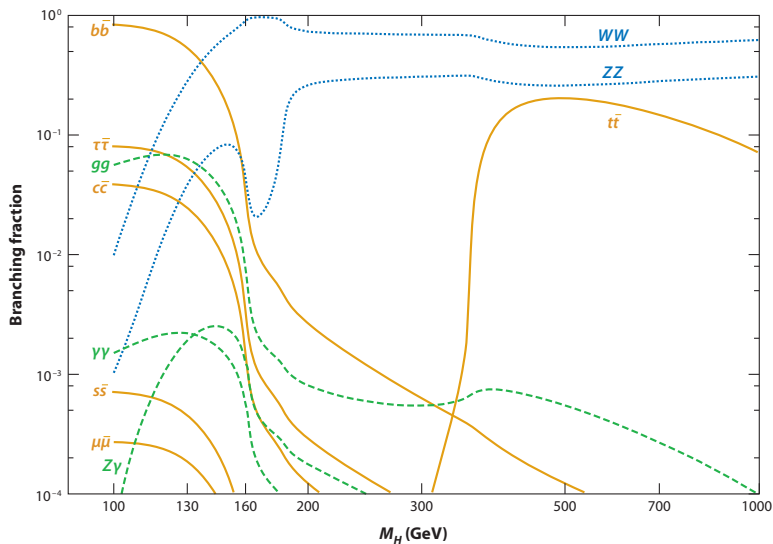
$$\Gamma(H \rightarrow Z^0 Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively

$2x^2$ and $2x'^2$ terms \Leftrightarrow decays into transverse gauge bosons

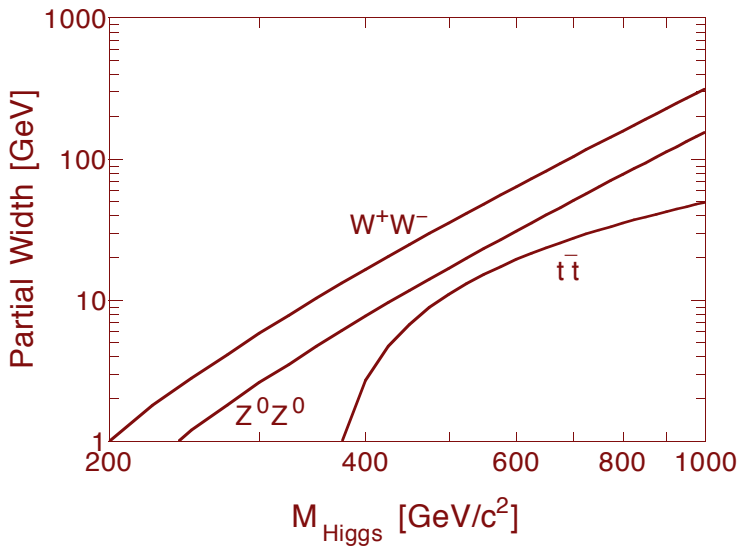
Dominant decays for large M_H : pairs of longitudinal weak bosons

SM Higgs Boson Branching Fractions



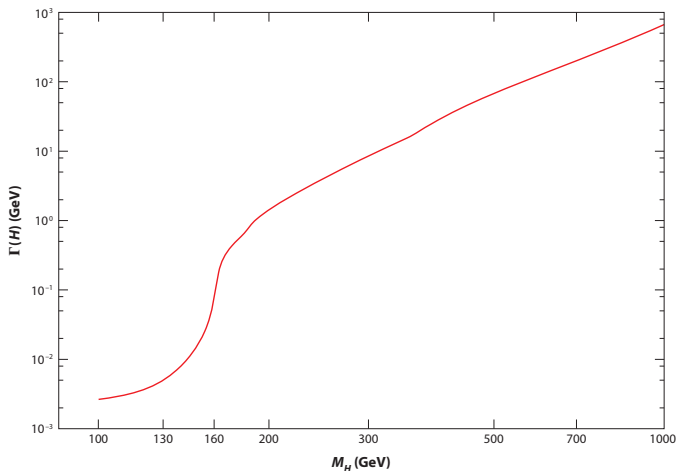
Djouadi, hep-ph/0503172

Dominant decays at high mass



For $M_H \rightarrow 1 \text{ TeV}$, Higgs boson is *ephemeral*: $\Gamma_H \rightarrow M_H$.

Total width of the standard-model Higgs boson



Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

A few words on Higgs production ...

$e^+e^- \rightarrow H$: hopelessly small

$\mu^+\mu^- \rightarrow H$: scaled by $(m_\mu/m_e)^2 \approx 40\,000$

$e^+e^- \rightarrow HZ$: prime channel

Hadron colliders:

$gg \rightarrow H \rightarrow b\bar{b}$: background ?!

$gg \rightarrow H \rightarrow \tau\tau, \gamma\gamma$: rate ?!

$gg \rightarrow H \rightarrow W^+W^-$: best Tevatron sensitivity now

$\bar{p}p \rightarrow H(W, Z)$: prime Tevatron channel for light Higgs

At the LHC:

Many channels accessible, search sensitive up to 1 TeV

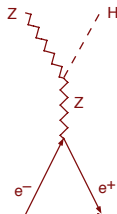
Higgs search in e^+e^- collisions

$\sigma(e^+e^- \rightarrow H \rightarrow \text{all})$ is *minute*, $\propto m_e^2$

Even narrowness of low-mass H is not enough to make it visible ... Sets aside a traditional strength of e^+e^- machines—*pole physics*

Most promising:

associated production $e^+e^- \rightarrow HZ$
(has no small couplings)

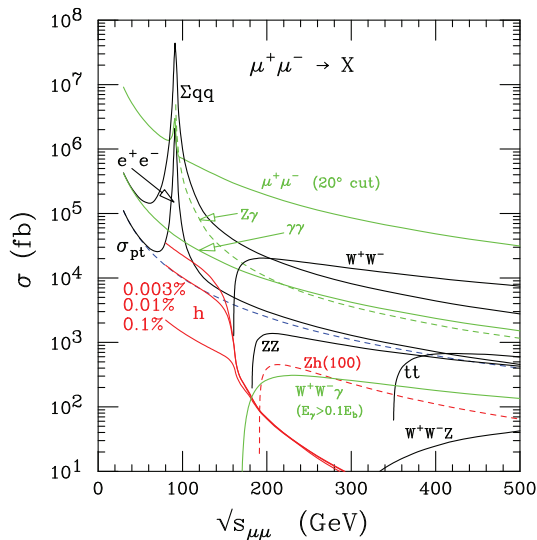


$$\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 x_W^2(1 - x_W)^2}$$

K : c.m. momentum of H

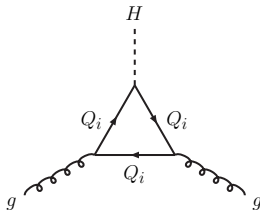
$x_W \equiv \sin^2 \theta_W$

$$l^+l^- \rightarrow X \dots$$



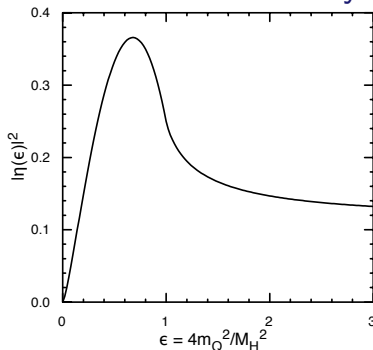
$$\sigma(e^+e^- \rightarrow H) = (m_e/m_\mu)^2 \sigma(\mu^+\mu^- \rightarrow H) \approx \sigma(\mu^+\mu^- \rightarrow H)/40\,000$$

H couples to gluons through quark loops

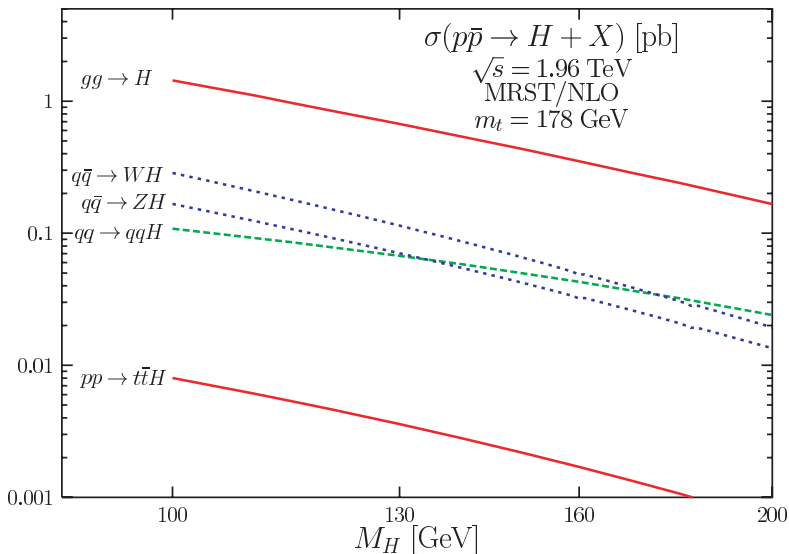


Only heavy quarks matter:

heavy 4th generation ??



Higgs-boson production at the Tevatron

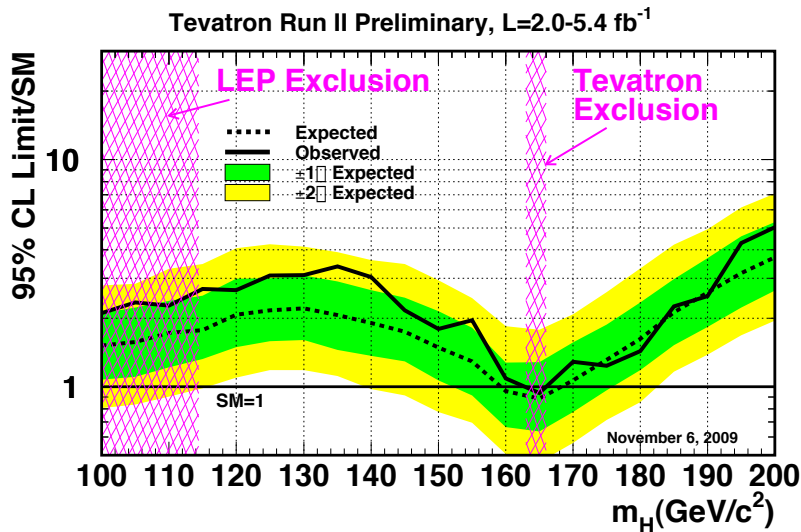


Djouadi

Update 1

Update 2

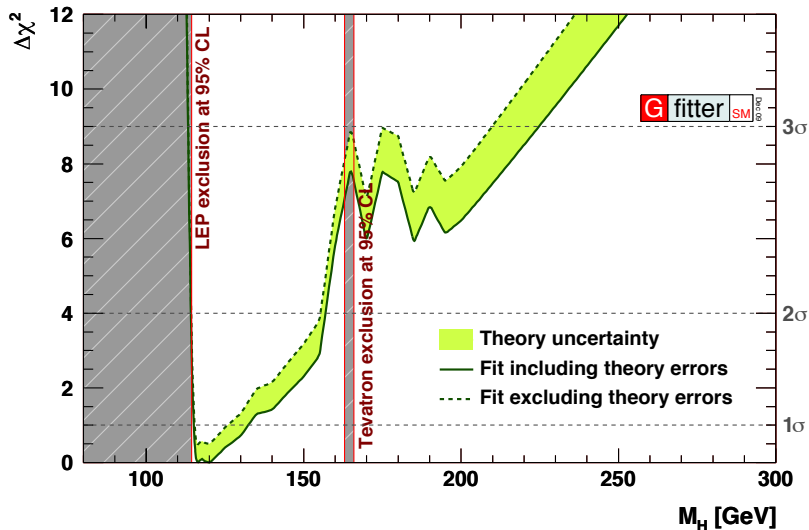
Current Tevatron Sensitivity



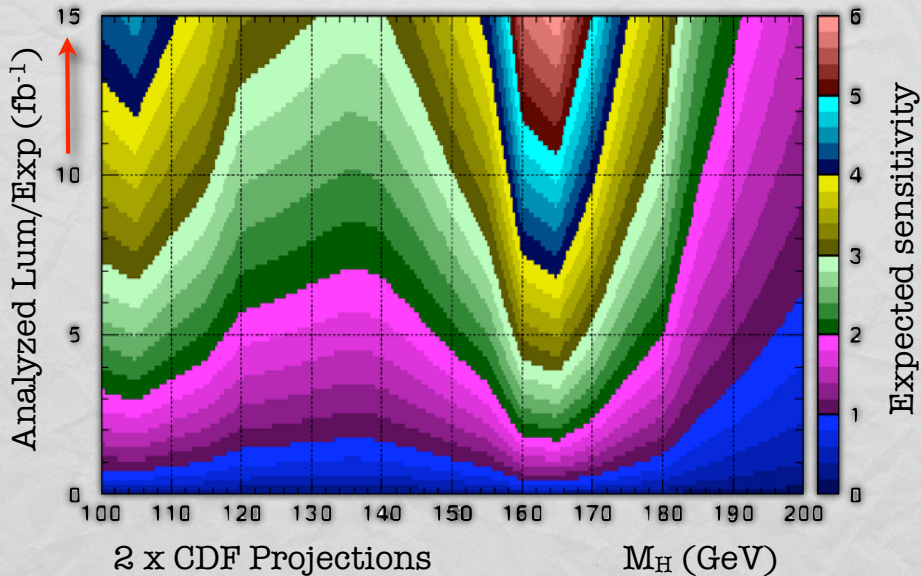
combining experiments, channels: Fall 2009

Electroweak theory projection

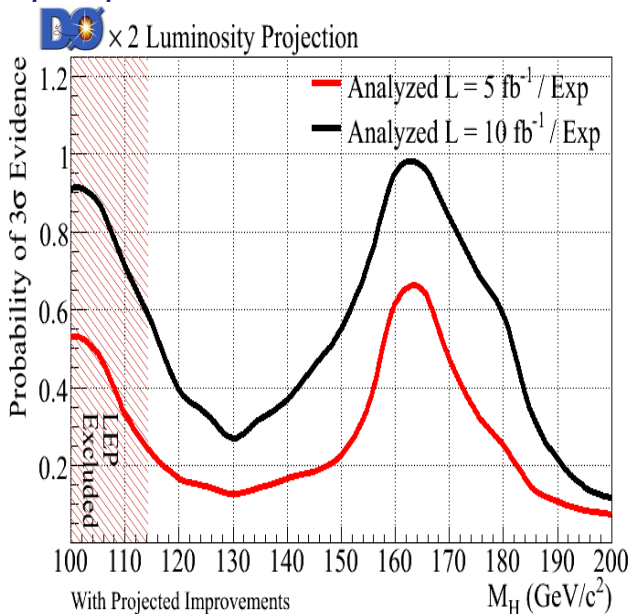
Global fit + exclusions



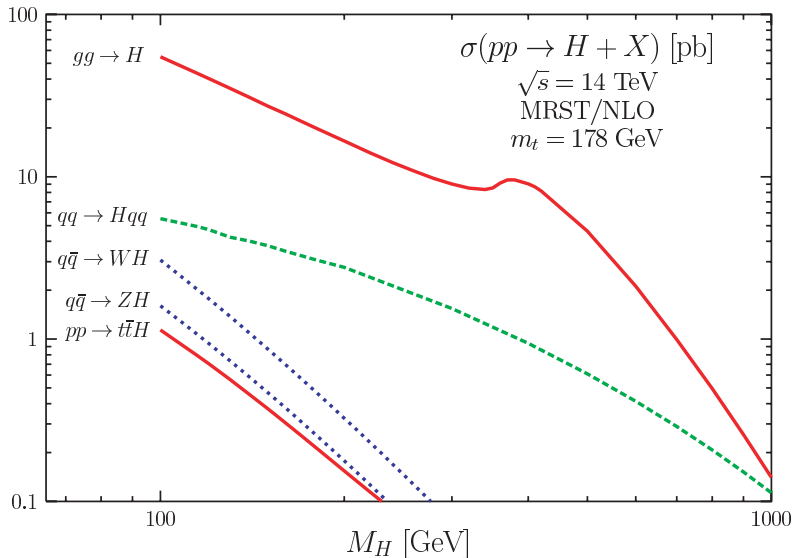
Tevatron prospects ... Konigsberg, La Thuile 2010



With projected improvements achieved



LHC cross sections ...



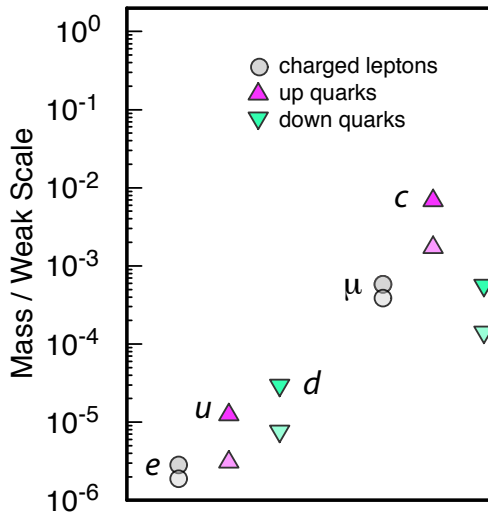
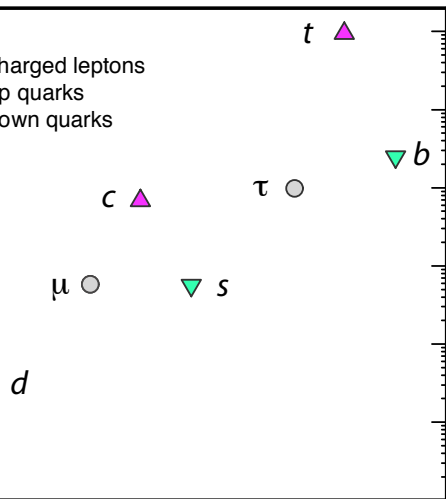
Djouadi

SM (electroweak theory) shortcomings

- No explanation of Higgs potential
- No prediction for M_H
- Doesn't predict fermion masses & mixings
- M_H unstable to quantum corrections
- No explanation of charge quantization
- Doesn't account for three generations
- Vacuum energy problem
- Beyond scope: dark matter, matter asymmetry, etc.

~> imagine more complete, predictive extensions

Fermion Mass Generation



Masses evolved to unification scale

Fermion mass is accommodated, not explained

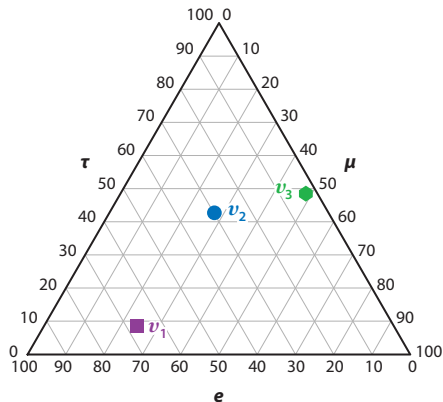
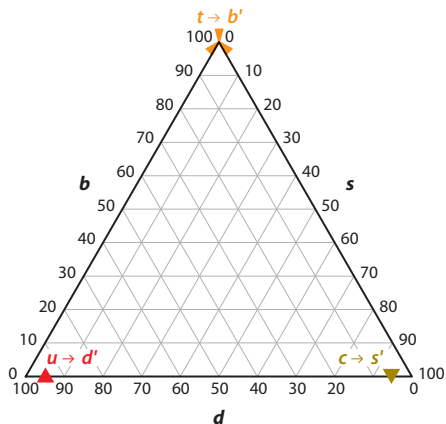
- All fermion masses \sim physics beyond the standard model!
- $\zeta_t \approx 1$ $\zeta_e \approx 3 \times 10^{-6}$ $\zeta_\nu \approx 10^{-11}$??

What accounts for the range and values of the Yukawa couplings?

- There may be *other sources* of neutrino mass

The Problem of Identity

Quark and Lepton Mixing



What makes a top quark a top quark, ...?

The Hierarchy Problem

Evolution of the Higgs-boson mass

$$M_H^2(p^2) = M_H^2(\Lambda^2) + \text{[triangle loop]} + \text{[bubble loop]} + \text{[sun loop]}$$


quantum corrections from particles with $J = 0, \frac{1}{2}, 1$

Potential divergences:

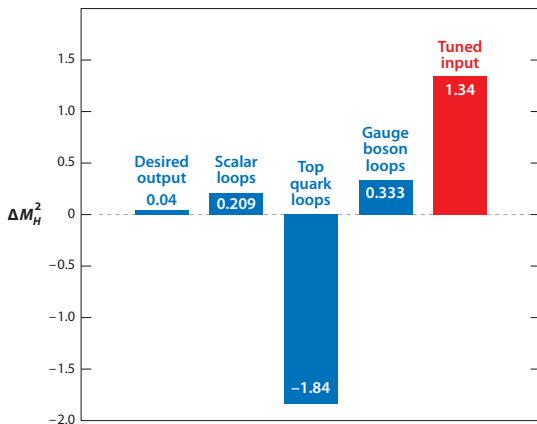
$$M_H^2(p^2) = M_H^2(\Lambda^2) + C g^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots,$$

Λ : naturally large, $\sim M_{\text{Planck}}$ or $\sim U \approx 10^{15-16}$ GeV

How to control quantum corrections?

A Delicate Balance ... even for $\Lambda = 5$ TeV

$$\delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2 \sqrt{2}} (6M_W^2 + 3M_Z^2 + M_H^2 - 12m_t^2)$$



Light Higgs + no new physics: LEP Paradox

The Hierarchy Problem

Possible paths

- Fine tuning
- A new symmetry (supersymmetry)
fermion, boson loops contribute with opposite sign
- Composite “Higgs boson” (technicolor . . .)
form factor damps integrand
- Little Higgs models, etc.
- Low-scale gravity (shortens range of integration)

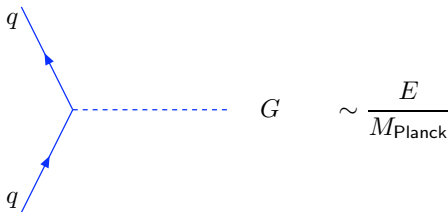
All but first require new physics near the TeV scale

Why is empty space so nearly massless?

Natural to neglect gravity in particle physics ...

Gravitational ep interaction $\approx 10^{-41} \times \text{EM}$

$$G_{\text{Newton}} \text{ small} \iff M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}} \right)^{\frac{1}{2}} \approx 1.22 \times 10^{19} \text{ GeV large}$$



$$\text{Estimate } B(K \rightarrow \pi G) \sim \left(\frac{M_K}{M_{\text{Planck}}} \right)^2 \sim 10^{-38}$$

But gravity is not always negligible ...

The vacuum energy problem

$$\text{Higgs potential } V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

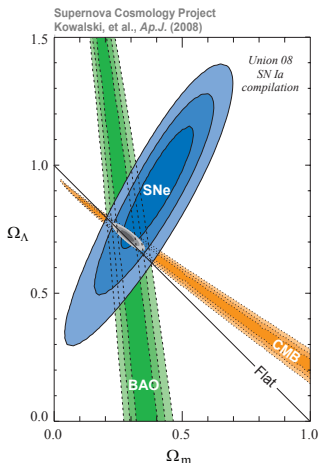
$V \neq 0$ contributes position-independent **vacuum energy density**

$$\rho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8 \text{ GeV}^4 \approx 10^{24} \text{ g cm}^{-3}$$

Adding vacuum energy density ρ_{vac} \Leftrightarrow adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad \Lambda = \frac{8\pi G_N}{c^4} \rho_{\text{vac}}$$

$$\rho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4 \approx \rho_{\text{crit}} = 3H_0^2/8\pi G_N$$



$\rho_H \gtrsim 10^8 \text{ GeV}^4$: mismatch by 10^{54}

A dull headache for thirty years ...

► H constraints

Stability bounds

Quantum corrections to $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$

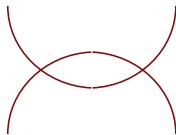
Triviality of scalar field theory bounds M_H from above

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- \Rightarrow *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

Bounding M_H from above ...

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

(Perturbation theory reliable only when λ is small,
lattice field theory treats strong-coupling regime)

Bounding M_H from above ...

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

...implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu)$$

Bounding M_H from above ...

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \rightarrow \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the **bound** forces $\lambda(\mu)$ to zero.

→ free field theory “trivial”

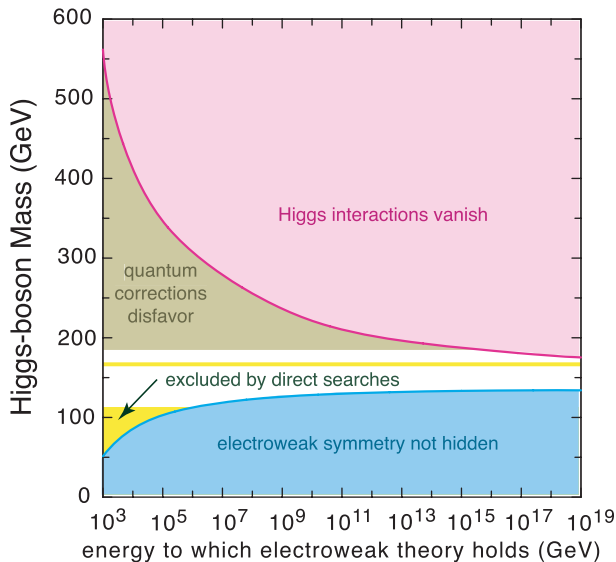
Rewrite as bound on M_H :

$$\Lambda \leq \mu \exp \left(\frac{2\pi^2}{3\lambda(\mu)} \right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp \left(4\pi^2 v^2 / 3M_H^2 \right)$$

Bounding M_H from above ...



Bounding M_H from above ...

Moral: For any M_H , there is a *maximum energy scale* Λ^* at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \rightarrow 1 \text{ TeV}/c^2$ and interactions become strong

Lattice analyses $\implies M_H \lesssim 710 \pm 60 \text{ GeV}$ if theory describes physics to a few percent up to a few TeV

If $M_H \rightarrow 1 \text{ TeV}$ EW theory lives on brink of instability

Requiring $V(v) < V(0)$ gives *lower* bound on M_H

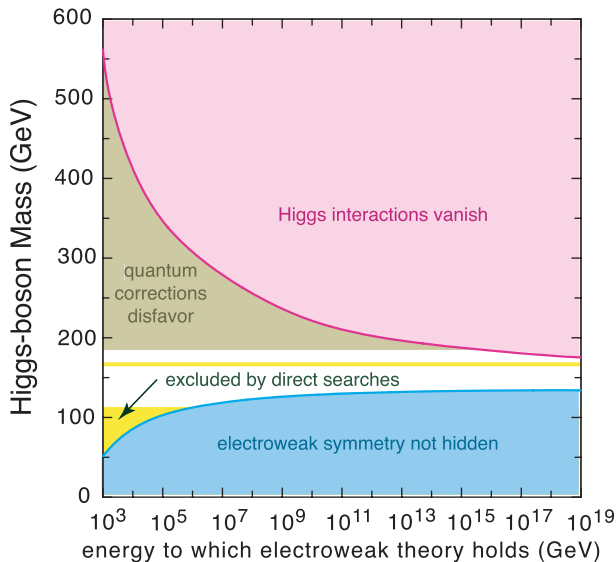
Requiring that $\langle \phi \rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition ... (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy m_t)

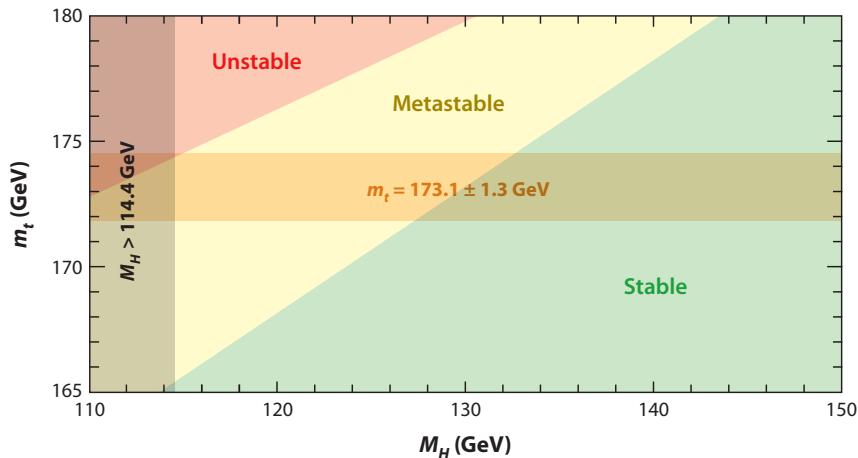
If Higgs boson is relatively light (which would require explanation) then theory can be self-consistent up to very high energies

Consistent to M_{Planck} if $134 \text{ GeV} \lesssim M_H \lesssim 177 \text{ GeV}$



Living on the Edge?

Require cosmological tunneling time, not absolute stability



Isidori, et al., hep-ph/0104016

LHC physics run has begun!

The Large Hadron Collider is running in 2010–2011 at 3.5 TeV per beam, to accumulate $\sim 1 \text{ fb}^{-1}$.

- How is the physics potential compromised by running below 14 TeV?
- At what point will the LHC begin to explore virgin territory and surpass the discovery reach of the Tevatron experiments CDF and D0?

[arXiv:0908.3660](https://arxiv.org/abs/0908.3660)

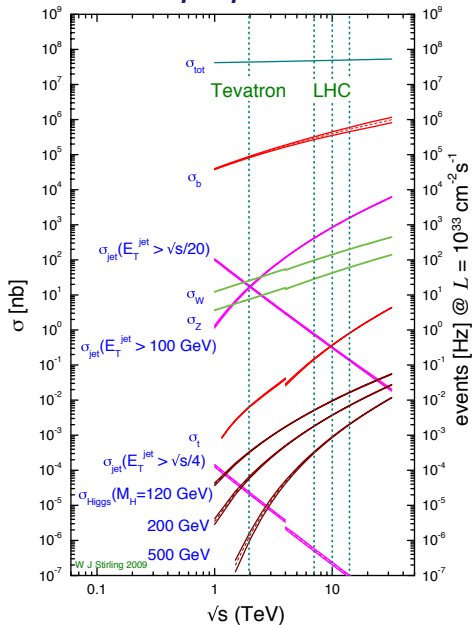
lutece.fnal.gov/PartonLum

EHLQ, *Rev. Mod. Phys.* **56**, 579–707 (1984)

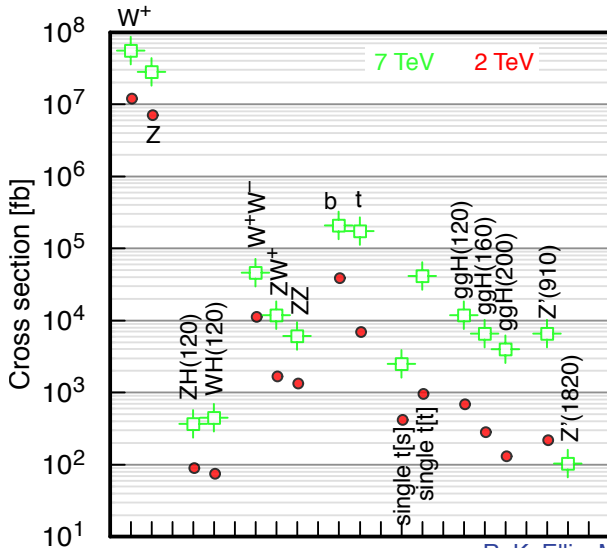
Ellis, Stirling, Webber, *QCD & Collider Physics*

MRSW08NLO examples + RKE Lecture 3, SUSSP 2009

Sample event rates in $p^\pm p$ collisions



Some Absolute Rates



R. K. Ellis, MCFM

What Is a Proton?

(For hard scattering) a broad-band, unselected beam of quarks, antiquarks, gluons, & perhaps other constituents, characterized by parton densities

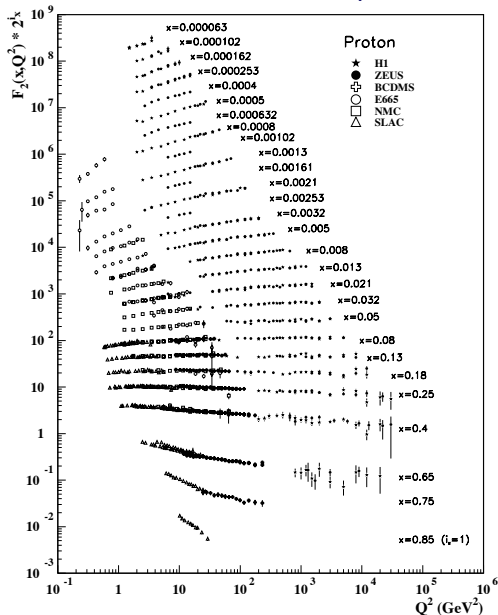
$$f_i^{(a)}(x_a, Q^2),$$

... number density of species i with momentum fraction x_a of hadron a seen by probe with resolving power Q^2 .

Q^2 evolution given by QCD perturbation theory

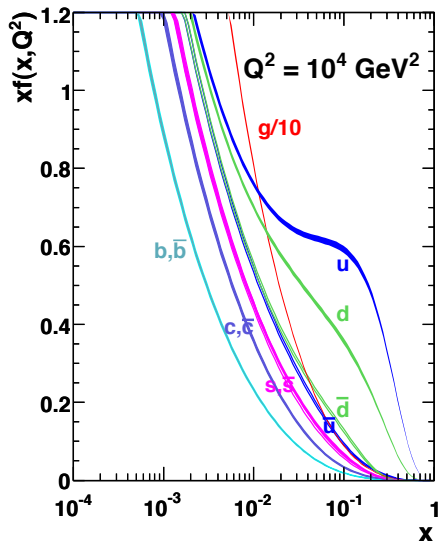
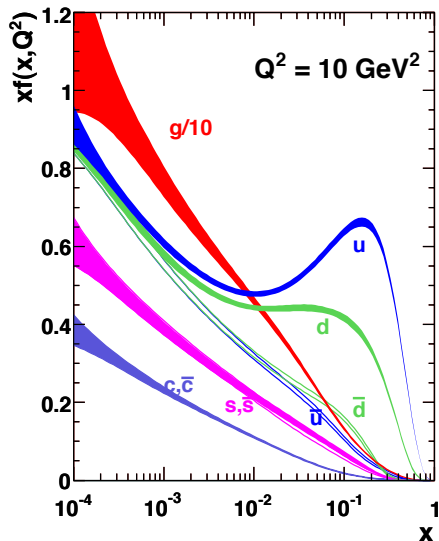
$$f_i^{(a)}(x_a, Q_0^2): \text{ nonperturbative}$$

Deeply Inelastic Scattering $\leadsto f_i^{(a)}(x_a, Q_0^2)$



What Is a Proton?

MSTW 2008 NLO PDFs (68% C.L.)



Hard-scattering cross sections

$$d\sigma(a + b \rightarrow c + X) = \sum_{ij} \int dx_a dx_b \cdot$$

$$f_i^{(a)}(x_a, Q^2) f_j^{(b)}(x_b, Q^2) d\hat{\sigma}(i + j \rightarrow c + X),$$

$d\hat{\sigma}$: elementary cross section at energy $\sqrt{\hat{s}} = \sqrt{x_a x_b s}$

Define differential luminosity ($\tau = \hat{s}/s$)

$$\frac{d\mathcal{L}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 dx \left[f_i^{(a)}(x) f_j^{(b)}(\tau/x) + f_j^{(a)}(x) f_i^{(b)}(\tau/x) \right]$$

parton i -parton j collisions in $(\tau, \tau + d\tau)$ per ab collision

$$d\sigma(a + b \rightarrow c + X) = \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(i + j \rightarrow c + X)$$

Parton Luminosities + Prior Knowledge = Answers

Hard scattering: $\hat{\sigma} \propto 1/\hat{s}$; Resonance: $\hat{\sigma} \propto \tau$; form

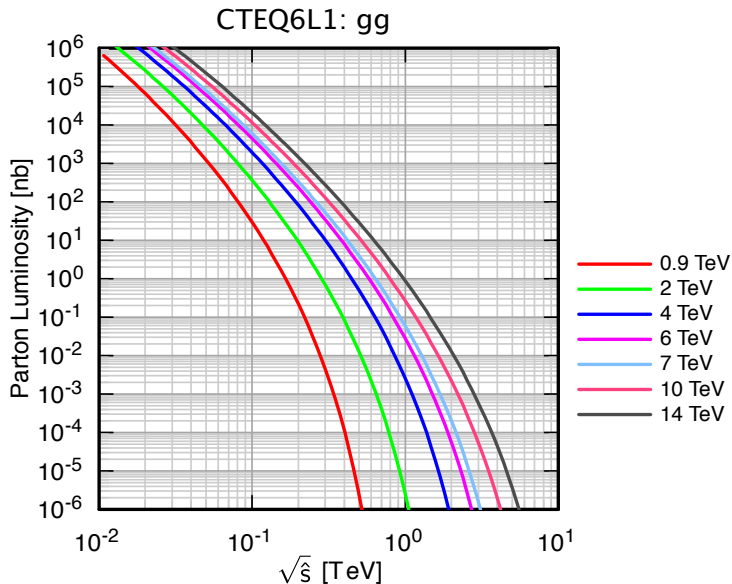
$$\frac{\tau}{\hat{s}} \frac{d\mathcal{L}}{d\tau} \equiv \frac{\tau/\hat{s}}{1 + \delta_{ij}} \int_{\tau}^1 \frac{dx}{x} [f_i^{(a)}(x) f_j^{(b)}(\tau/x) + f_j^{(a)}(x) f_i^{(b)}(\tau/x)]$$

(convenient measure of parton ij luminosity)

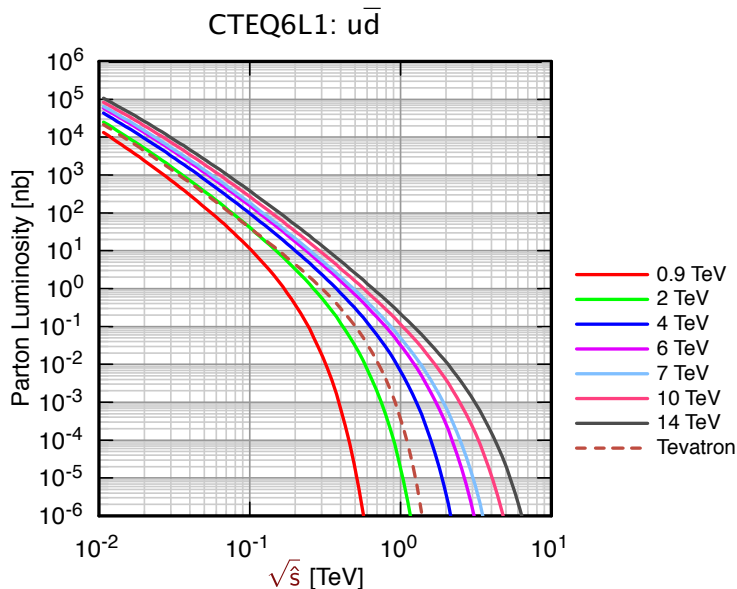
$$f_i^{(a)}(x): \text{pdf}; \quad \tau = \hat{s}/s$$

$$\sigma(s) = \sum_{\{ij\}} \int_{\tau_0}^1 \frac{d\tau}{\tau} \cdot \frac{\tau}{\hat{s}} \frac{d\mathcal{L}_{ij}}{d\tau} \cdot [\hat{s} \hat{\sigma}_{ij}(\hat{s})]$$

Parton Luminosity

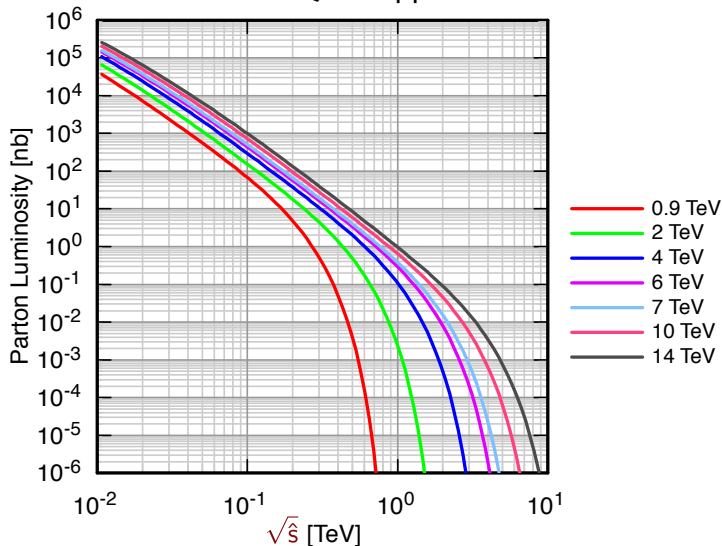


Parton Luminosity

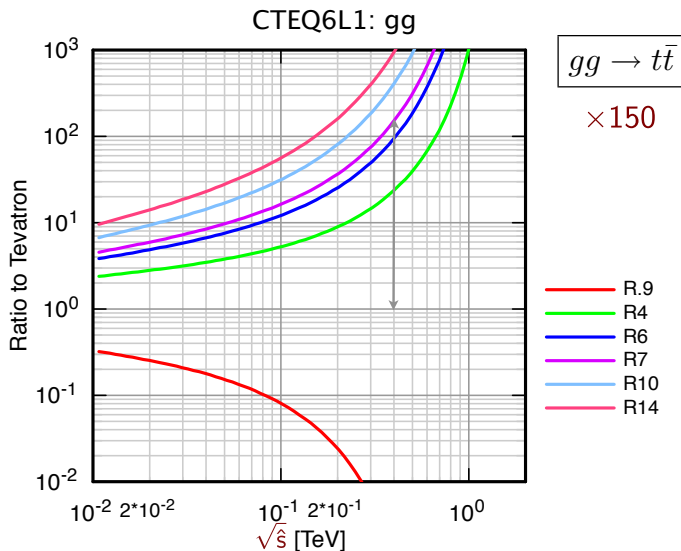


Parton Luminosity (light quarks)

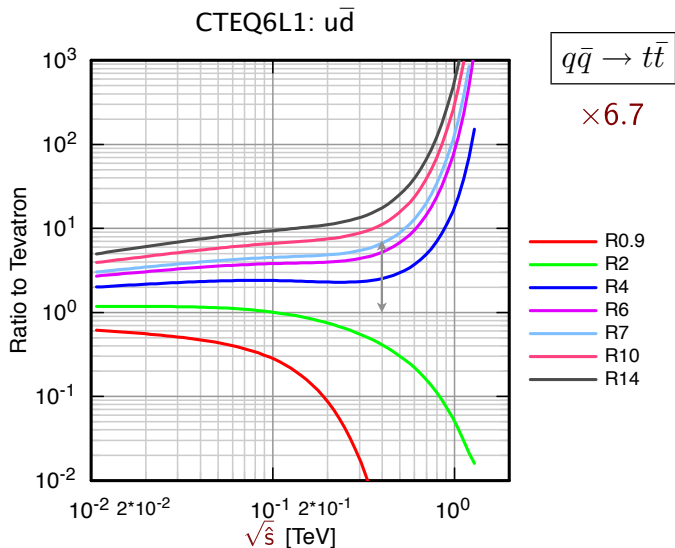
CTEQ6L1: qq



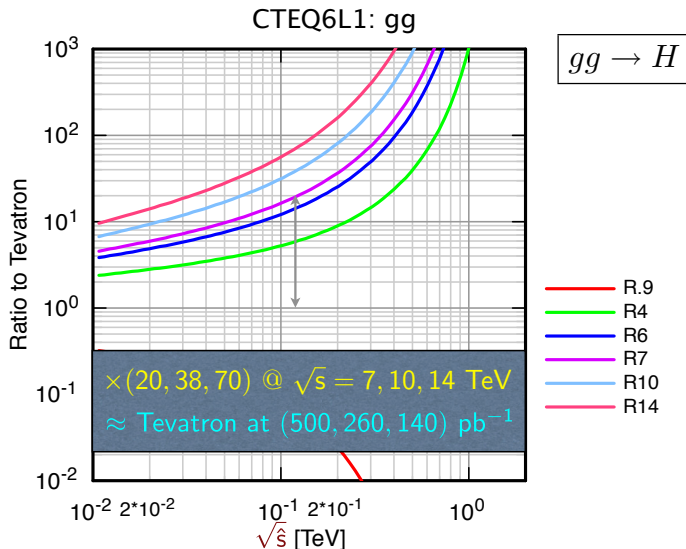
Luminosity Ratios



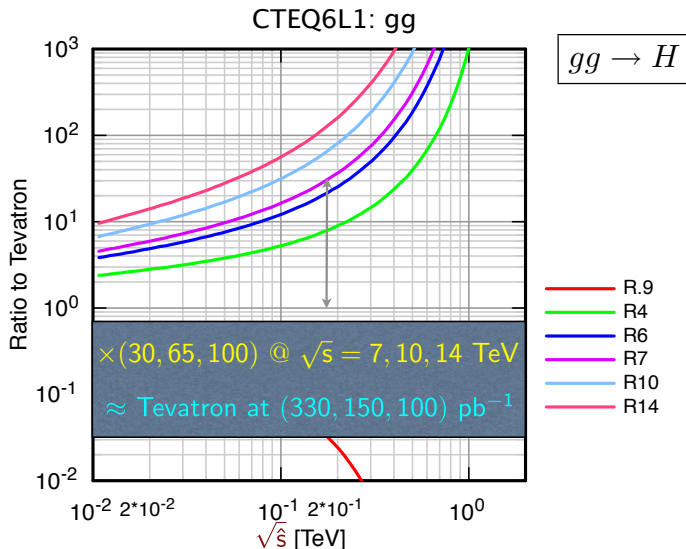
Luminosity Ratios



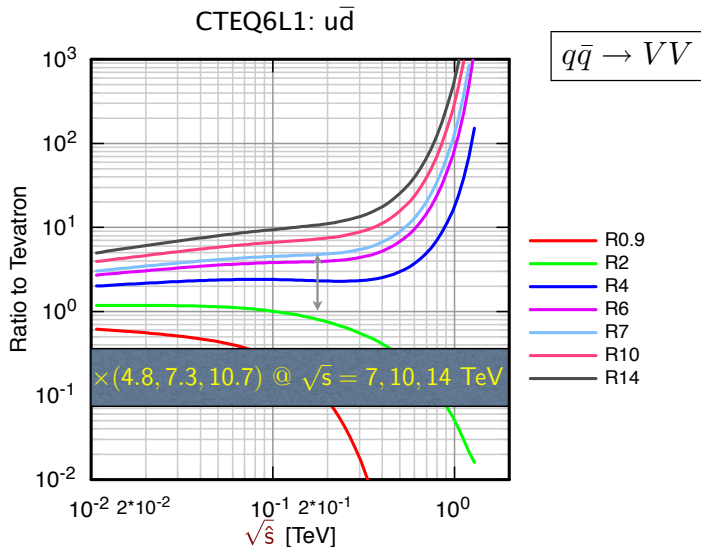
Luminosity Ratios



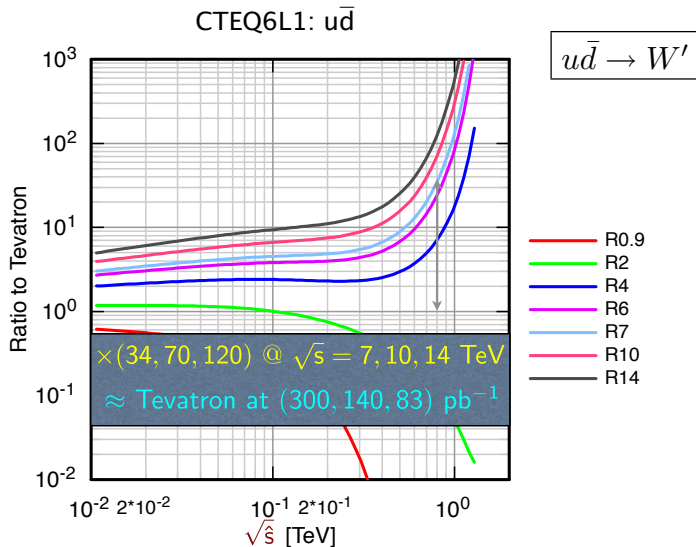
Luminosity Ratios



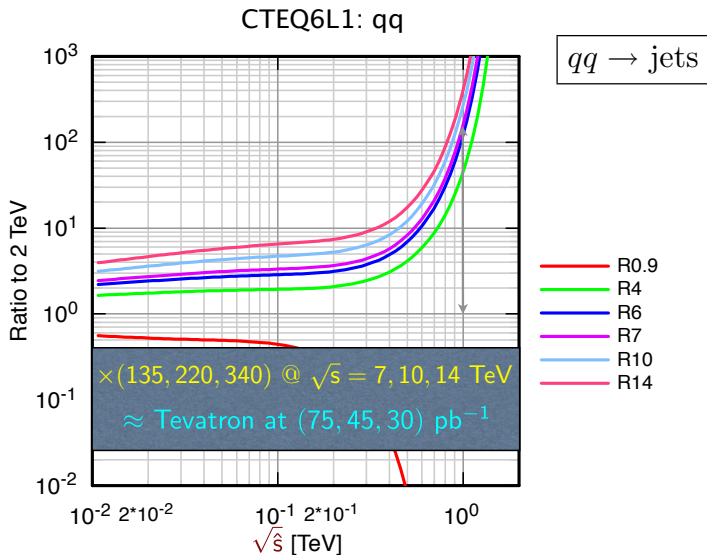
Luminosity Ratios



Luminosity Ratios

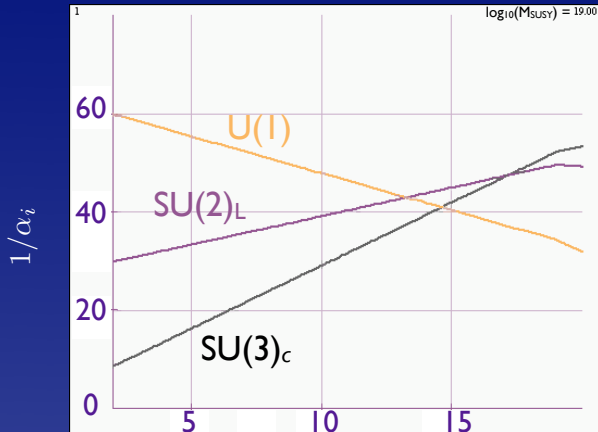


Luminosity Ratios



Coupling Constant Unification

Different running of $U(1)_Y$, $SU(2)_L$, $SU(3)_c$
gives possibility of coupling constant unification

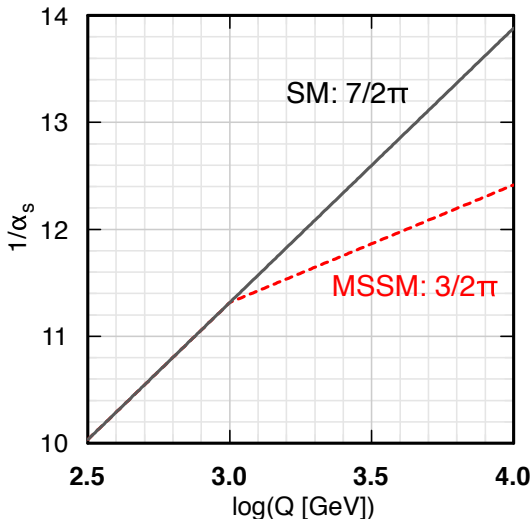


$$\alpha^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$

$$\log_{10}(E[\text{GeV}])$$

Can LHC See Change in Evolution?

Sensitive to new colored particles



(sharp threshold illustrated)

... also for $\sin^2 \theta_W$

Why Electroweak Symmetry Breaking Matters

PHYSICAL REVIEW D **79**, 096002 (2009)

Gedanken worlds without Higgs fields: QCD-induced electroweak symmetry breaking

Chris Quigg^{1,2} and Robert Shrock³

¹*Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*

²*Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*

³*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*
(Received 29 January 2009; published 4 May 2009)

To illuminate how electroweak symmetry breaking shapes the physical world, we investigate toy models in which no Higgs fields or other constructs are introduced to induce spontaneous symmetry breaking. Two models incorporate the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry and fermion content similar to that of the standard model. The first class—like the standard electroweak theory—contains no bare mass terms, so the spontaneous breaking of chiral symmetry within quantum chromodynamics is the only source of electroweak symmetry breaking. The second class adds bare fermion masses sufficiently small that QCD remains the dominant source of electroweak symmetry breaking and the model can serve as a well-behaved low-energy effective field theory to energies somewhat above the hadronic scale. A third class of models is based on the left-right-symmetric $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge group. In a fourth class of models, built on $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ gauge symmetry, the lepton number is treated as a fourth color and the color gauge group is enlarged to the $SU(4)_{PS}$ of Pati and Salam (PS). Many interesting characteristics of the models stem from the fact that the effective strength of the weak interactions is much closer to that of the residual strong interactions than in the real world. The Higgs-free models not only provide informative contrasts to the real world, but also lead us to consider intriguing issues in the application of field theory to the real world.

DOI: [10.1103/PhysRevD.79.096002](https://doi.org/10.1103/PhysRevD.79.096002)

PACS numbers: 11.15.-q, 12.10.-g, 12.60.-i

Challenge: Understanding the Everyday World

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

(No EWSB agent at $v \approx 246$ GeV)

Consider effects of **all** SM interactions!

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Modified Standard Model: No Higgs Sector: $\overline{\text{SM}}_1$

$\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$ with massless u, d, e, ν

(treat $\text{SU}(2)_L \otimes \text{U}(1)_Y$ as perturbation)

Nucleon mass little changed:

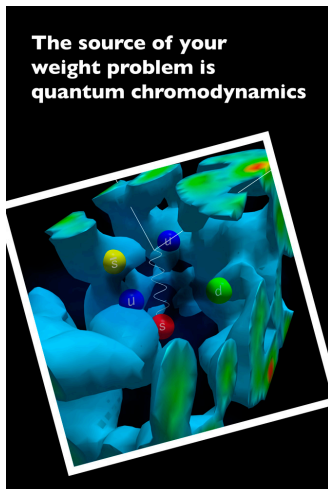
$$M_p = C \cdot \Lambda_{\text{QCD}} + \dots$$

$$3 \frac{m_u + m_d}{2} = (7.5 \text{ to } 15) \text{ MeV}$$

Small contribution from virtual strange quarks

M_N decreases by $< 10\%$ in chiral limit: $939 \rightsquigarrow 870 \text{ MeV}$

QCD accounts for (most) visible mass in Universe



(not the Higgs boson)

Modified Standard Model: No Higgs Sector: $\overline{\text{SM}}_1$

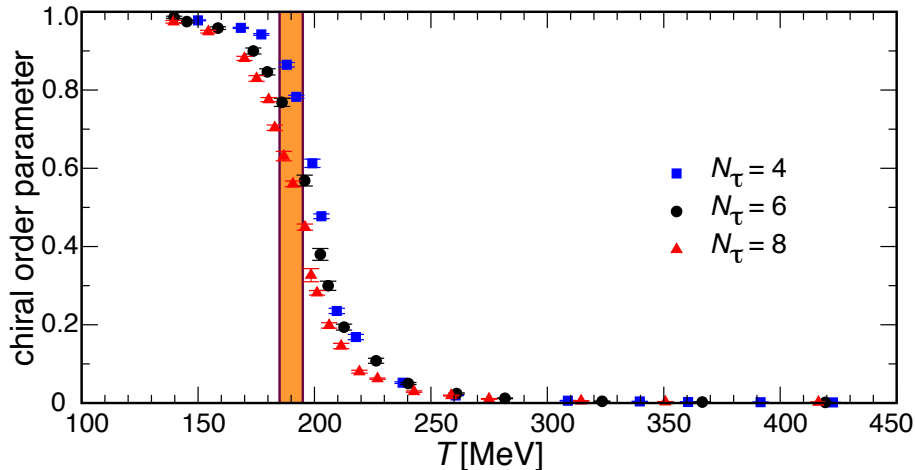
QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry.

At an energy scale $\sim \Lambda_{\text{QCD}}$, strong interactions become strong, fermion condensates $\langle \bar{q}q \rangle$ appear, and

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$

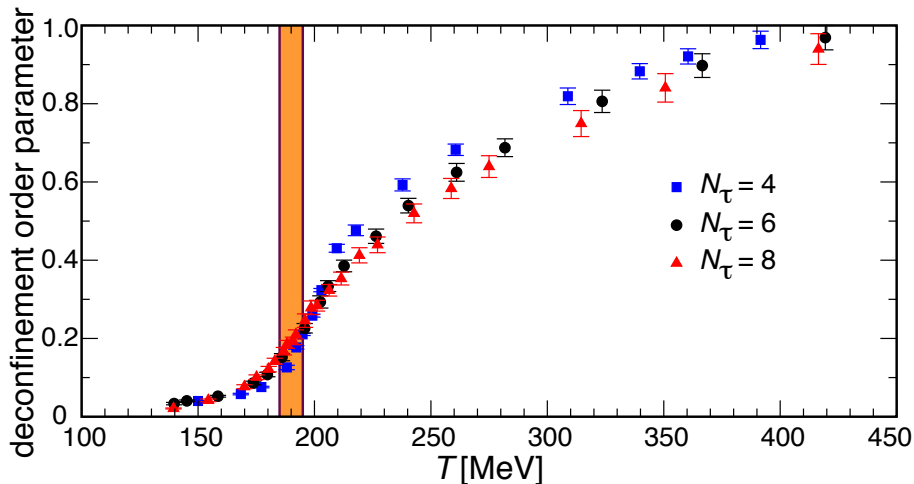
\leadsto 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

Chiral Symmetry Breaking on the Lattice



Weise lecture for review and lattice QCD references

Deconfinement on the Lattice



A. Polyakov, *Phys. Lett.* **B72**, 477 (1978)

Fermion condensate ...

links left-handed, right-handed fermions

$$\langle \bar{q} q \rangle = \langle \bar{q}_R q_L + \bar{q}_L q_R \rangle$$

$$1 = \frac{1}{2}(1 + \gamma_5) + \frac{1}{2}(1 - \gamma_5)$$

$$Q_L^a = \begin{pmatrix} u^a \\ d^a \end{pmatrix}_L \quad u_R^a \quad d_R^a$$

$$(\text{SU}(3)_c, \text{SU}(2)_L)_Y: (\mathbf{3}, \mathbf{2})_{1/3} \quad (\mathbf{3}, \mathbf{1})_{4/3} \quad (\mathbf{3}, \mathbf{1})_{-2/3}$$

transforms as $\text{SU}(2)_L$ doublet with $|Y| = 1$

Induced breaking of $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$

Broken generators: 3 axial currents; couplings to π : \bar{f}_π

Turn on $SU(2)_L \otimes U(1)_Y$:

Weak bosons couple to axial currents, acquire mass $\sim g\bar{f}_\pi$

$$g \approx 0.65, g' \approx 0.34, f_\pi = 92.4 \text{ MeV} \rightsquigarrow \bar{f}_\pi \approx 87 \text{ MeV}$$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{\bar{f}_\pi^2}{4} \quad (w_1, w_2, w_3, \mathcal{A})$$

same structure as standard EW theory

Induced breaking of $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$

Diagonalize:

$$\overline{M}_W^2 = g^2 \bar{f}_\pi^2 / 4$$

$$\overline{M}_Z^2 = (g^2 + g'^2) \bar{f}_\pi^2 / 4$$

$$\overline{M}_A^2 = 0$$

$$\overline{M}_Z^2 / \overline{M}_W^2 = (g^2 + g'^2) / g^2 = 1 / \cos^2 \theta_W$$

NGBs become longitudinal components of weak bosons.

$$\overline{M}_W \approx 28 \text{ MeV}$$

$$\overline{M}_Z \approx 32 \text{ MeV}$$

$$(M_W \approx 80 \text{ GeV}$$

$$M_Z \approx 91 \text{ GeV})$$

No fermion masses ...

(Possible division of labor)

Inspiration for Technicolor \rightsquigarrow Extended Technicolor ...

Higher scales? $uu \rightarrow X^{4/3} \rightarrow e^+ d^c$ mixes p, e^+

$$\varepsilon \equiv \mathcal{M}(p \leftrightarrow e^+) \approx \frac{4\pi\alpha_U}{M_X^2} \Lambda_{\text{QCD}}^3 \approx 10^{-36} \text{ GeV}$$

(e^+, p) mass matrix

$$M = \begin{pmatrix} 0 & \varepsilon \\ \varepsilon^* & M_p \end{pmatrix}$$

$$\rightsquigarrow m_e = |\varepsilon|^2 / M_p \approx 10^{-72} \text{ GeV}$$

Electroweak scale

EW theory: *choose* $v = (G_F \sqrt{2})^{-1/2} \approx 246 \text{ GeV}$

$\overline{\text{SM}}$: *predict*

$$\overline{G}_F = 1/(\overline{f}_\pi^2 \sqrt{2}) \approx 93.25 \text{ GeV}^{-2} \approx 8 \times 10^6 G_F$$

Cross sections, decay rates $\times (\overline{G}_F / G_F)^2 \approx 6.4 \times 10^{13}$

Real world: $\sigma(\nu_e n \rightarrow e^- p) \approx 10^{-38} \text{ cm}^{-2}$

$\leadsto \overline{\text{SM}}$: $\bar{\sigma}(\nu_e n \rightarrow e^- p) \approx \text{few mb}$

Weak interaction strength \sim residual strong interactions

$\overline{\text{SM}}_1$: Hadron Spectrum

Pions absent (became longitudinal W^\pm, Z^0)

ρ, ω, a_1 “as usual,” but

$$\rho^0 \rightarrow W^+ W^-$$

$$\rho^+ \rightarrow W^+ Z$$

$$\omega \rightarrow W^+ W^- Z$$

$$M_\Delta > M_N; \quad \Delta \rightarrow N(W^\pm, Z, \gamma)$$

Nucleon mass little changed: look in detail

Nucleon masses ...

“Obvious” that proton should outweigh neutron

...but false in real world: $M_n - M_p \approx 1.293 \text{ MeV}$

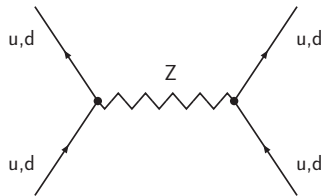
Real-world contributions,

$$\begin{aligned} M_n - M_p &= (m_d - m_u) \cancel{(m_d - m_u)} - \frac{1}{3} (\delta m_q + \delta M_C + \delta M) \\ &\leadsto -1.7 \text{ MeV} \end{aligned}$$

...but weak contributions enter.

Weak contributions are not negligible

$$\overline{M}_n - \overline{M}_p|_{\text{weak}} \propto dd - uu$$



$$\begin{aligned} \overline{M}_n - \overline{M}_p|_{\text{weak}} &= \frac{\overline{G}_F \Lambda_h^3 \sqrt{2}}{3} x_W (1 - 2x_W) \approx \frac{\overline{G}_F \Lambda_h^3 \sqrt{2}}{24} \\ &= \frac{\Lambda_h^3}{3\overline{f}_\pi^2} x_W (1 - 2x_W) \approx \frac{\Lambda_h^3}{24\overline{f}_\pi^2} > 0 \end{aligned}$$

$$x_W = \sin^2 \theta_W \approx \frac{1}{4}$$

perhaps a few MeV?

Consequences for β decay

Scale decay rate $\Gamma \sim \overline{G}_F^2 |\overline{\Delta M}|^5 / 192\pi^3$ (rapid!)

$$\bar{\tau}_\mu \rightarrow 10^{-19} \text{ s}$$

$$n \rightarrow p e^- \bar{\nu}_e \text{ or } p \rightarrow n e^+ \nu_e$$

Example: $|\overline{M}_n - \overline{M}_p| = M_n - M_p \rightsquigarrow \bar{\tau}_N \approx 14 \text{ ps}$

No Hydrogen Atom?

Neutron could be lightest nucleus

Strong coupling in $\overline{\text{SM}}$

In SM, Higgs boson regulates high-energy behavior

Gedanken experiment: scattering of

$$W_L^+ W_L^- \quad \frac{Z_L^0 Z_L^0}{\sqrt{2}} \quad \frac{HH}{\sqrt{2}} \quad HZ_L^0$$

In high-energy limit, s -wave amplitudes

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}.$$

Strong coupling in $\overline{\text{SM}}$

In *standard model*, $|a_0| \leq 1$ yields

$$M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 4v\sqrt{\pi/3} = 1 \text{ TeV}$$

In $\overline{\text{SM}}_1$ *Gedanken* world,

$$\overline{M}_H \leq \left(\frac{8\pi\sqrt{2}}{3\overline{G}_F} \right)^{1/2} = 4\overline{f}_\pi\sqrt{\pi/3} \approx 350 \text{ MeV}$$

violated because no Higgs boson \leadsto strong scattering

Strong coupling in $\overline{\text{SM}}$

SM with (very) heavy Higgs boson:

s -wave W^+W^- , Z^0Z^0 scattering as $s \gg M_W^2, M_Z^2$:

$$a_0 = \frac{s}{32\pi v^2} \begin{bmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{bmatrix}$$

Largest eigenvalue: $a_0^{\text{max}} = s/16\pi v^2$

$$|a_0| \leq 1 \Rightarrow \sqrt{s^*} = 4\sqrt{\pi}v \approx 1.74 \text{ TeV}$$

$$\overline{\text{SM}}: \sqrt{s^*} = 4\sqrt{\pi}\bar{f}_\pi \approx 620 \text{ MeV}$$

$\overline{\text{SM}}$ becomes strongly coupled on the hadronic scale

Strong coupling in $\overline{\text{SM}}$

As in standard model ...

$I = 0, J = 0$ and $I = 1, J = 1$: attractive

$I = 2, J = 0$: repulsive

As partial-wave amplitudes approach bounds,
 WW , WZ , ZZ resonances form,
multiple production of W and Z

in emulation of $\pi\pi$ scattering approaching 1 GeV

Detailed projections depend on unitarization protocol

What about atoms?

Suppose some light elements produced in BBN survive

Massless $e \implies \infty$ Bohr radius

No meaningful atoms

No valence bonding

No integrity of matter, no stable structures

Massless fermion pathologies ...

Vacuum readily breaks down to e^+e^- plasma

... persists with GUT-induced tiny masses

“hard” fermion masses: explicit $SU(2)_L \otimes U(1)_Y$ breaking
NGBs \longrightarrow pNGBs

$$\text{SM}m: a_J(f\bar{f} \rightarrow W_L^+ W_L^-) \propto G_F m_f E_{\text{cm}}$$

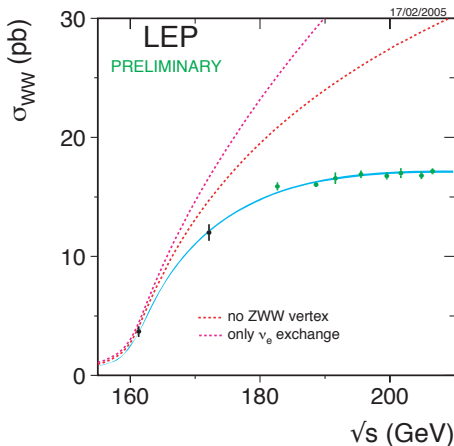
saturate p.w. unitarity at

$$\sqrt{s_f} \simeq \frac{4\pi\sqrt{2}}{\sqrt{3\eta_f} G_F m_f} = \frac{8\pi v^2}{\sqrt{3\eta_f} m_f}$$

$$\eta_f = 1(N_c) \text{ for leptons (quarks)}$$

Hard electron mass: $\sqrt{s_e} \approx 1.7 \times 10^9$ GeV ...

Gauge cancellation need not imply renormalizable theory



Hard top mass: $\sqrt{s_t} \approx 3$ TeV

Add explicit fermion masses to $\overline{\text{SM}}$: $\rightsquigarrow \overline{\text{SM}}_m$

$a_J(f\bar{f} \rightarrow W_L^+ W_L^-)$ unitarity respected up to

$$\sqrt{s^*} = 4\sqrt{\pi n_g} \bar{f}_\pi \approx 620\sqrt{n_g} \text{ MeV}$$

(condition from WW scattering)

$$\rightsquigarrow m_f \lesssim \frac{2\sqrt{\pi n_g} \bar{f}_\pi}{\sqrt{3\eta_f}} \approx \begin{cases} 126 \sqrt{n_g} \text{ MeV (leptons)} \\ 73 \sqrt{n_g} \text{ MeV (quarks)} \end{cases}$$

would accommodate real-world e , u , d masses

In summary ...

- $\overline{\text{SM}}$: QCD-induced $\text{SU}(2)_L \otimes \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}}$
- No fermion masses; division of labor?
- No physical pions in $\overline{\text{SM}}_1$
- No quark masses: might proton outweigh neutron?
- Infinitesimal m_e : integrity of matter compromised
- $\overline{\text{SM}}$ exhibits strong W, Z dynamics below 1 GeV
- $\overline{M}_W \approx 30 \text{ MeV}$ in *Gedanken* world
- $\overline{G}_F \sim 10^7 G_F$: accelerates β decay
- Weak, hadronic int. comparable; nuclear forces
- Infinitesimal m_ℓ : vacuum breakdown, e^+e^- plasma
- $\overline{\text{SM}}m$: effective theory through hadronic scale

Outlook

How different a world, without a Higgs mechanism:
preparation for interpreting experimental insights

$\overline{\text{SM}}$, $\overline{\text{SM}}m$: explicit theoretical laboratories
complement to studies that retain Higgs, vary v
(very intricate alternative realities)

Fresh look at the way we have understood the real world
(possibly > 1 source of SSB, “hard” fermion masses)

How might EWSB deviate from the Higgs mechanism?

Flavor physics . . .

may be where we see, or diagnose, the break in the SM

Some opportunities (see Buras, Flavour Theory: 2009)

- CKM matrix from tree-level decays (LHC**b**)
- $\mathcal{B}(B_{s,d} \rightarrow \mu^+ \mu^-)$
- $D^0 - \bar{D}^0$ mixing; CP violation
- FCNC in top decay: $t \rightarrow (c, u)\ell^+\ell^-$, etc.
- Correlate virtual effects with direct detection of new particles to test identification
- Tevatron experiments demonstrate capacity for very precise measurements: e.g., B_s mixing.

Electroweak Questions for the LHC. II

- New physics in pattern of Higgs-boson decays?
- Will (unexpected or rare) decays of H reveal new kinds of matter?
- What would discovery of > 1 Higgs boson imply?
- What stabilizes M_H below 1 TeV?
- How can a light H coexist with absence of new phenomena?
- Is EWSB emergent, connected with strong dynamics?
- Is EWSB related to gravity through extra spacetime dimensions?
- If new strong dynamics, how can we diagnose? What takes place of H ?

Thank you!

Good luck!